Numerical Investigation of Precessional Fishbone Nonlinear Dynamic : Impact of the nonlinear MHD effects

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Summary

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I. Precessional Fishbone Instability

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II. Theoretical Model and Linear Benchmark

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III. Fishbone Saturation

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III. Fishbone Saturation

IV. MHD Nonlinear Effects Impact on Fishbone Saturation

Precessional Fishbone

- Energetic Particles (EP) excite a MHD wave.

[McGuire, et al., 1983
Coppi, et al., 1986
White, et al. 1989]

- Features :
  - Most unstable mode : $m, n = 1,1$
  - Frequency of the toroidal precession velocity of the EP, ω_D

- EP generated by :
  - Heating systems : NBI, ICRH,….
  - Alpha particles

Energetic Particles : Deeply Trapped Particles

- Particles trapped in the low field side
- Banana orbit : $v_\perp \gg v_\parallel$
- $\omega_{cyclotron} \gg \omega_{bounce} \gg \omega_D$

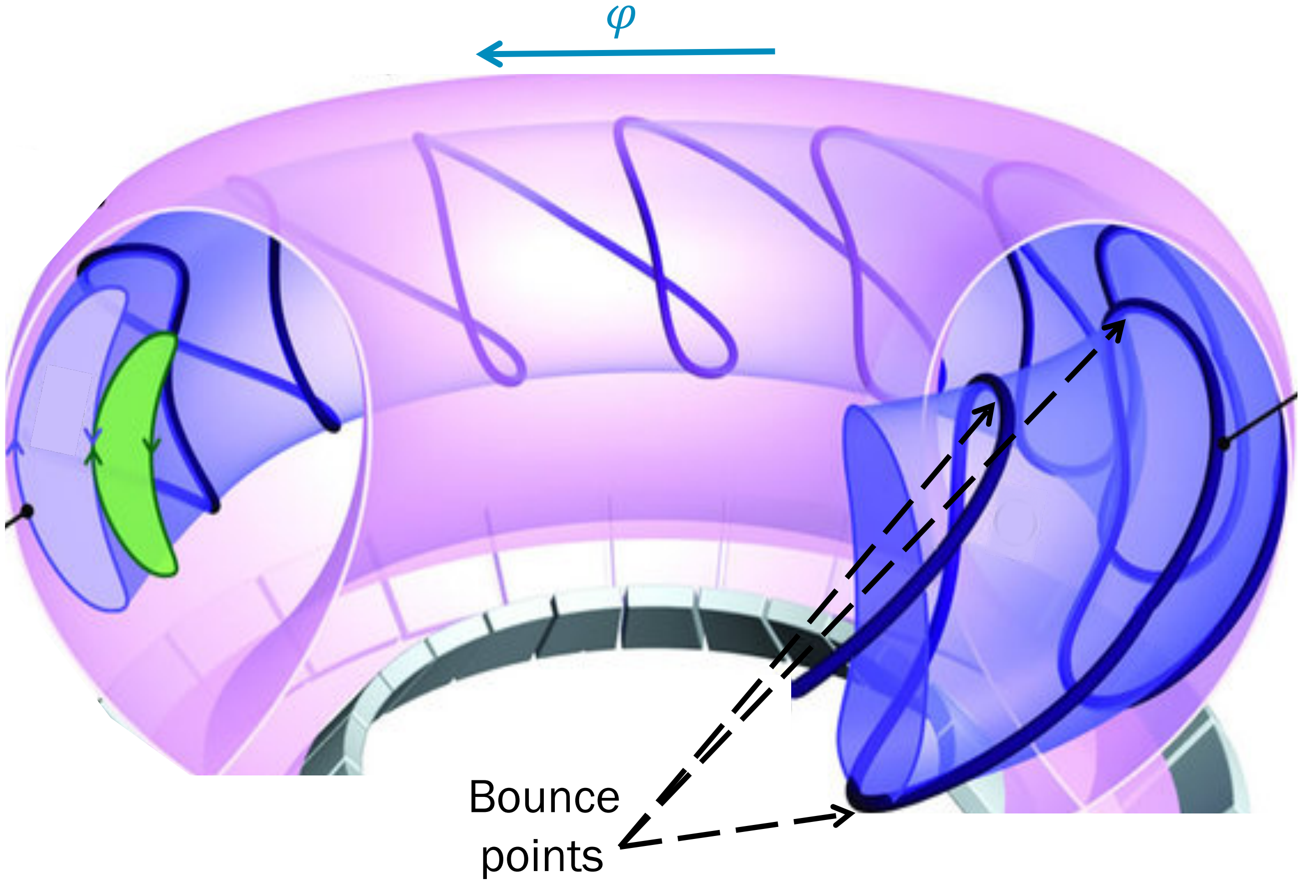


Illustration of the banana orbit in a tokamak,
Source : EUROfusion

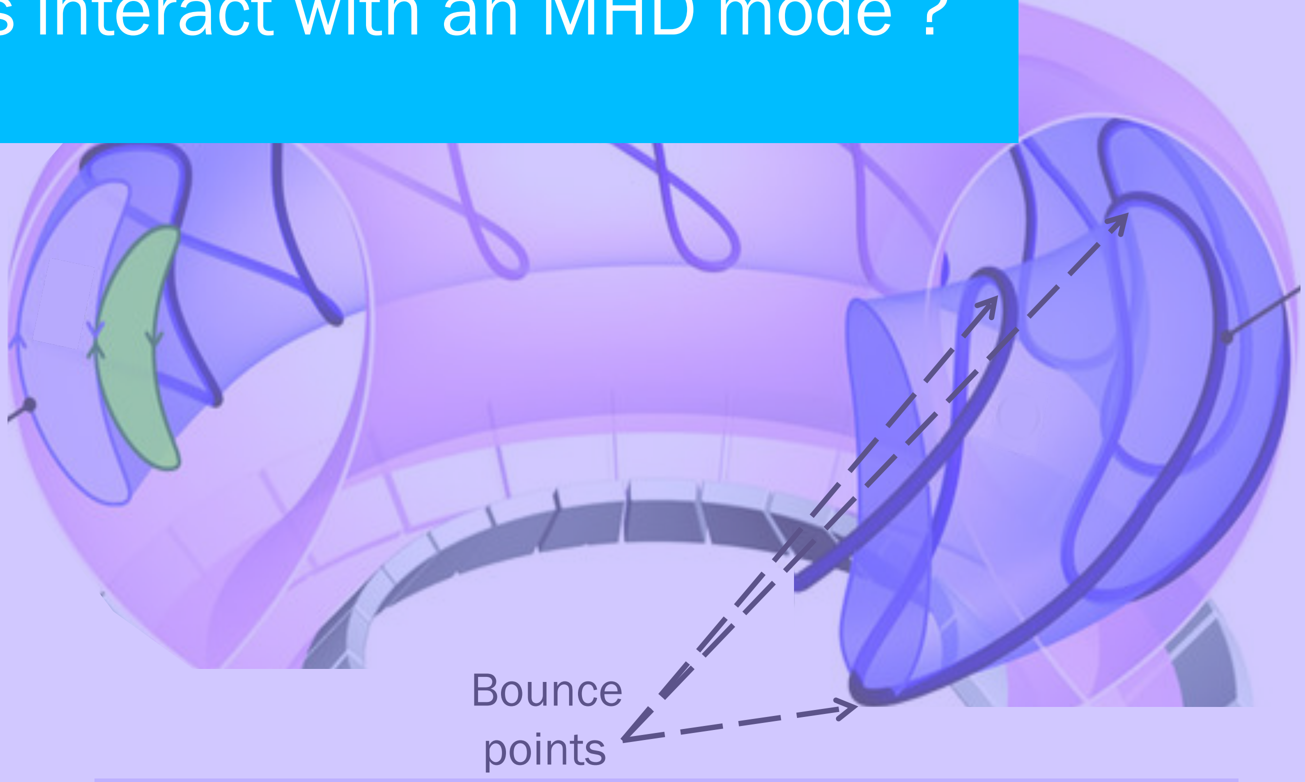
Energetic Particles : Deeply Trapped Particles

- Particle[partially covered]

How do Energetic Particles interact with an MHD mode ?

- Banana orbit : $v_{\perp} \gg v_{\parallel}$

- $\omega_{cyclotron} \gg \omega_{bounce} \gg \omega_D$



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Source : EUROfusion

Energetic Particles : Deeply Trapped Particles

- Particles
- Banana orbit : $v_\perp \gg v_\parallel$
- $\omega_{cyclotron}$

How do Energetic Particles interact with an MHD mode ?

Wave-particle resonance :

low frequency ω_D $\approx$ low frequency MHD mode

[Landau, et al., 1946
Bernstein, et al., 1957]

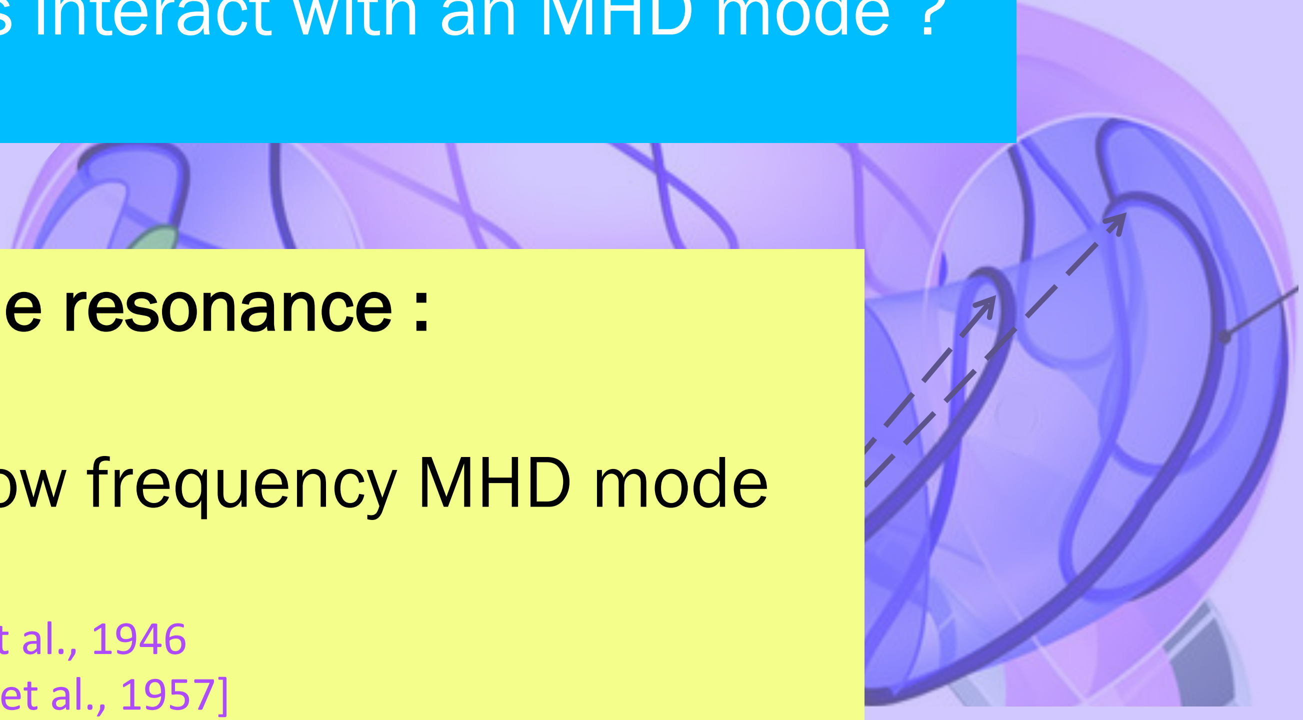

Illustration of the banana orbit in a tokamak,
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EP Kinetic Description

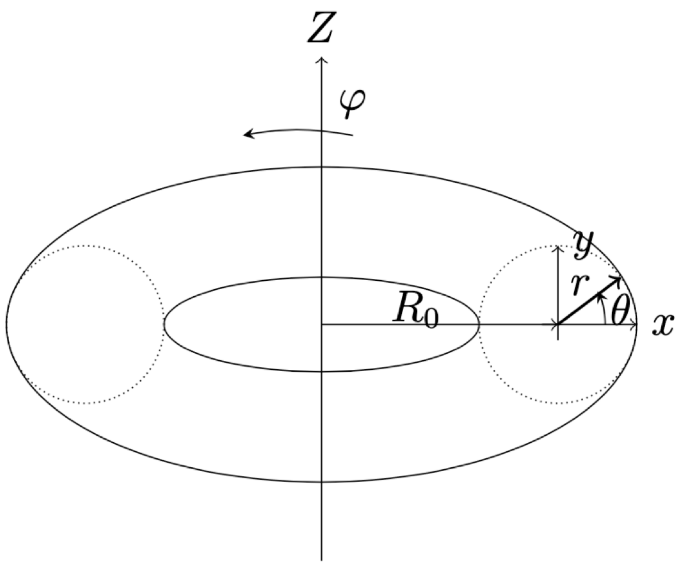
- Kinetic Vlasov equation :

$$\frac{\partial \mathrm{f}}{\partial \mathrm{t}} - \{\,\mathrm{H}, \mathrm{f}\,\}_{\varphi, p_\varphi} = 0$$

$$\mathrm{H} = \mu \mathrm{B}_\varphi + e\Phi$$

  - Φ is the bulk plasma electric potential
  - Toroidal precessional frequency :

$$\omega_D(r) = \frac{\mu}{e\,\mathrm{R}_0^2}\frac{\mathrm{B}_\varphi}{\mathrm{B}_{\mathrm{poloidal}}} \propto \frac{1}{r}$$



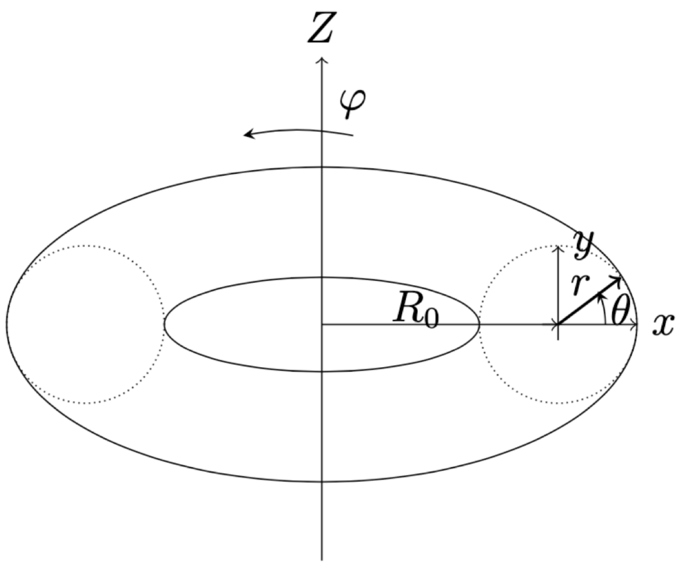
EP Kinetic Description

- Gyro average : 6D → 4D

  ▹ $\mu = \frac{m v_\perp^2}{2\,B_\varphi} = cst$, adiabatic invariant since $\omega \ll \omega_{cyclotron}$

- Bounce average : 4D → 2D

  ▹ $\delta_b = cst \ll 1$, adiabatic invariant since $\omega \ll \omega_{bounce}$



$$f_{\mu,\delta_b}(\underbrace{\varphi, p_\varphi}_{\text{2D phase space}})$$

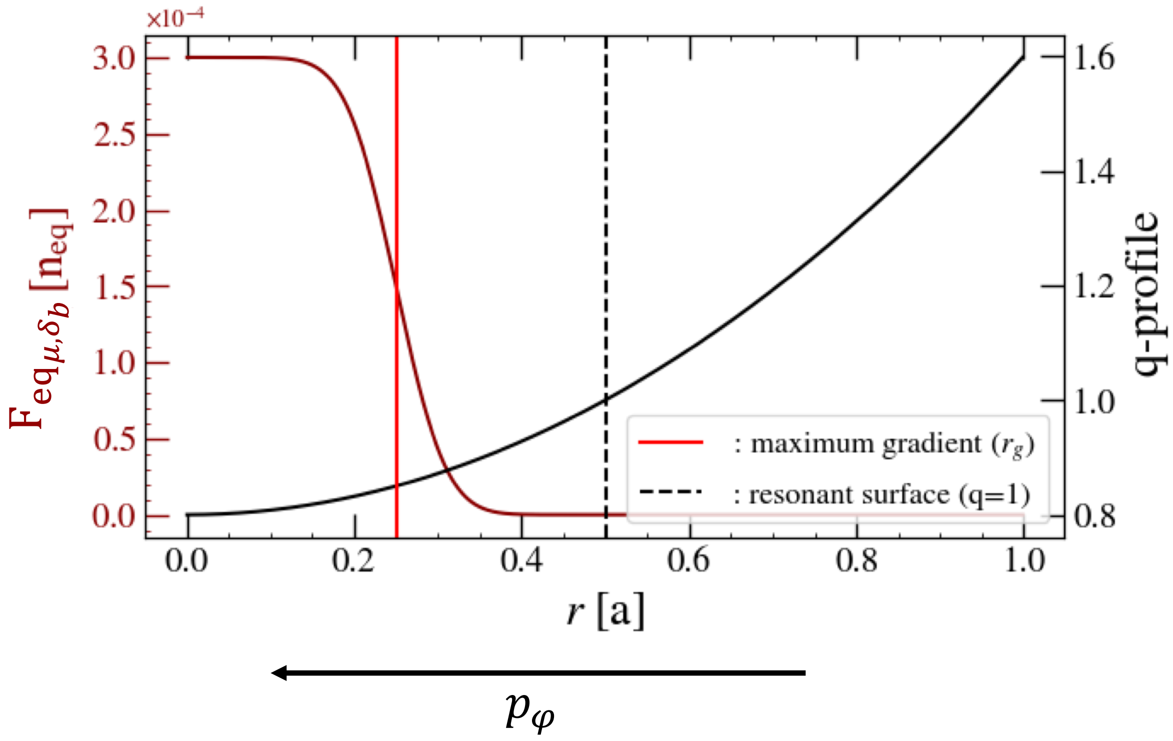
- $p_\varphi = m R_0 v_\parallel - e R_0 \Psi(r) \simeq -e R_0 \Psi(r)$ with $\Psi = A_\parallel$, parallel component of the vector potential

Gradient in r implies a gradient in p_φ

Resonant Interaction Mechanism

- EP density distribution function :

$$\mathrm{F}_{\mathrm{eq}_{\mu,\delta_b}} = \mathrm{A}\,\mathrm{F}_{\mathrm{r}}(\mathrm{r})$$



- Kinetic Destabilization :

$$\underbrace{\frac{\partial \mathrm{F}_{\mathrm{eq}}}{\partial p_\varphi}}_{>0} \Rightarrow \text{ Destabilization}$$

Inverse Landau Damping

At the frequency $\omega_D(r_g)$

[Landau, et al., 1946
Bernstein, et al., 1957]

Bulk Plasma Reduced MHD Description

- Advection of magnetic flux equation and vorticity equation :

$$\begin{cases} \dfrac{\partial}{\partial t}\Psi + \{\Phi, \Psi\} = \eta\Delta\Psi \\ \dfrac{\partial}{\partial t}\Delta\Phi + \{\Phi, \Delta\Phi\} = -\{\Psi, \Delta\Psi\} \end{cases} \xrightarrow[\text{At the } q=1 \text{ surface}]{\text{Destabilization of resistive internal kink}} \omega = 0, \quad \gamma \sim \eta^{\frac{1}{3}}$$

[Coppi, et al., 1976, White review, 1980]

Φ : Velocity stream function

$$\mathbf{u}_\perp = \frac{e_\varphi \times \nabla\Phi}{B}$$

Ψ : Parallel component of the potential vector

$$\mathbf{B}_\perp = e_\varphi \times \nabla\Psi$$

Bulk Plasma with EP Contribution

- Advection of magnetic flux equation and vorticity equation :

$$\begin{cases} \dfrac{\partial}{\partial t}\Psi + \{\Phi, \Psi\} = \eta\Delta\Psi \\ \dfrac{\partial}{\partial t}\Delta\Phi + \{\Phi, \Delta\Phi\} = -\{\Psi, \Delta\Psi\} \; -(e_\varphi \times \kappa).\nabla \boxed{P_{EP_\perp}} \end{cases}$$

Φ : Velocity stream function

$$\mathbf{u}_\perp = \frac{e_\varphi \times \nabla\Phi}{B}$$

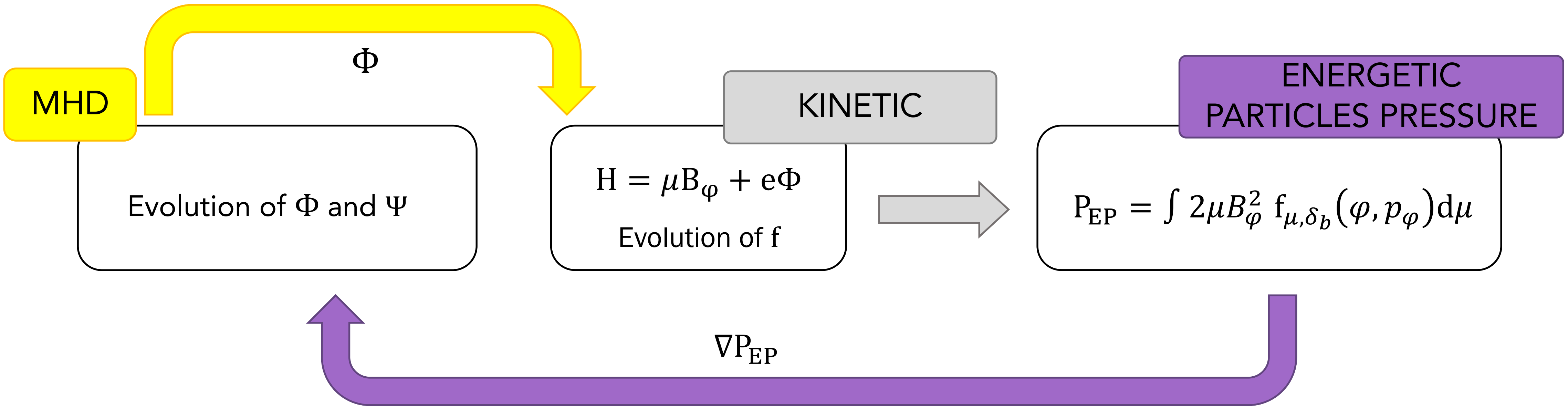
$\boxed{P_{EP_\perp}}$: EP Perpendicular pressure

$$P_{EP_\perp} = \int 2\mu B_\varphi^2 \; f_{\mu,\delta_b}(\varphi, p_\varphi) d\mu$$

Ψ : Parallel component of the potential vector

$$\mathbf{B}_\perp = e_\varphi \times \nabla\Psi$$

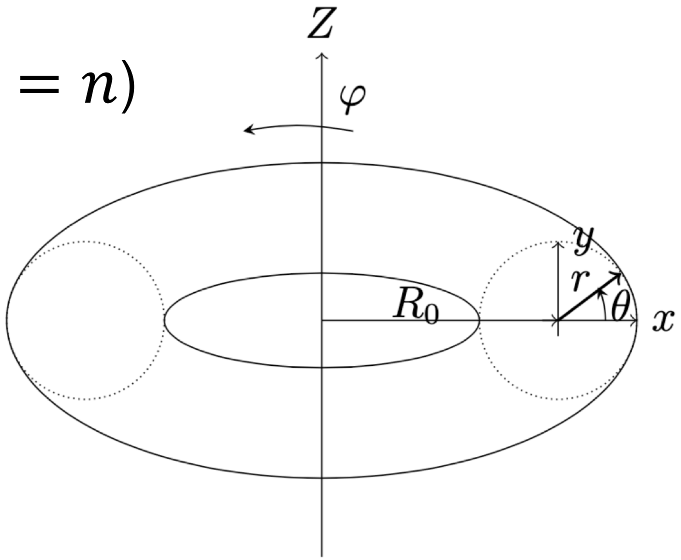
Bulk Plasma with EP Contribution



Simulation Setup

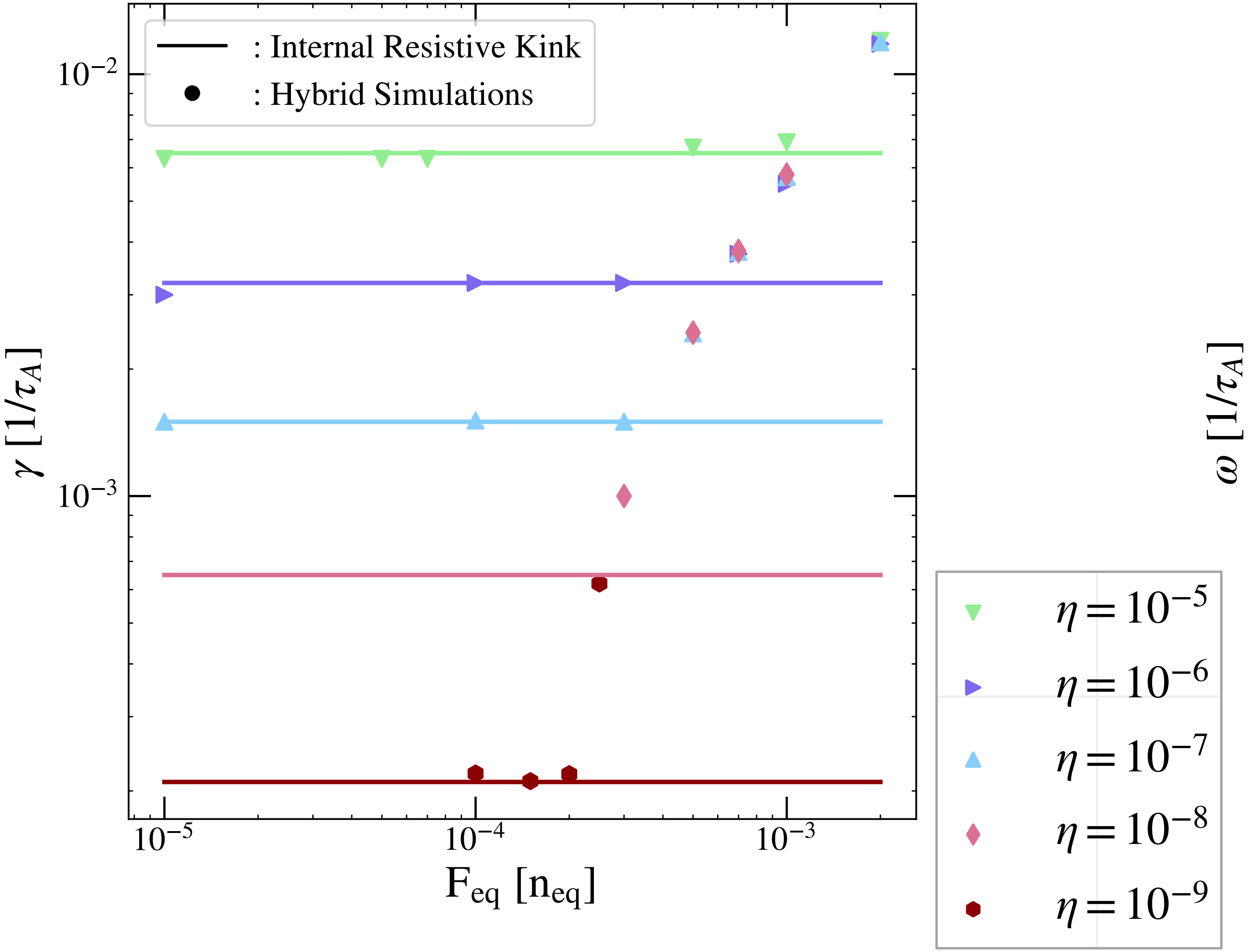
- Code : AMON [Semi-spectral code] [Poye, et al. 2014]

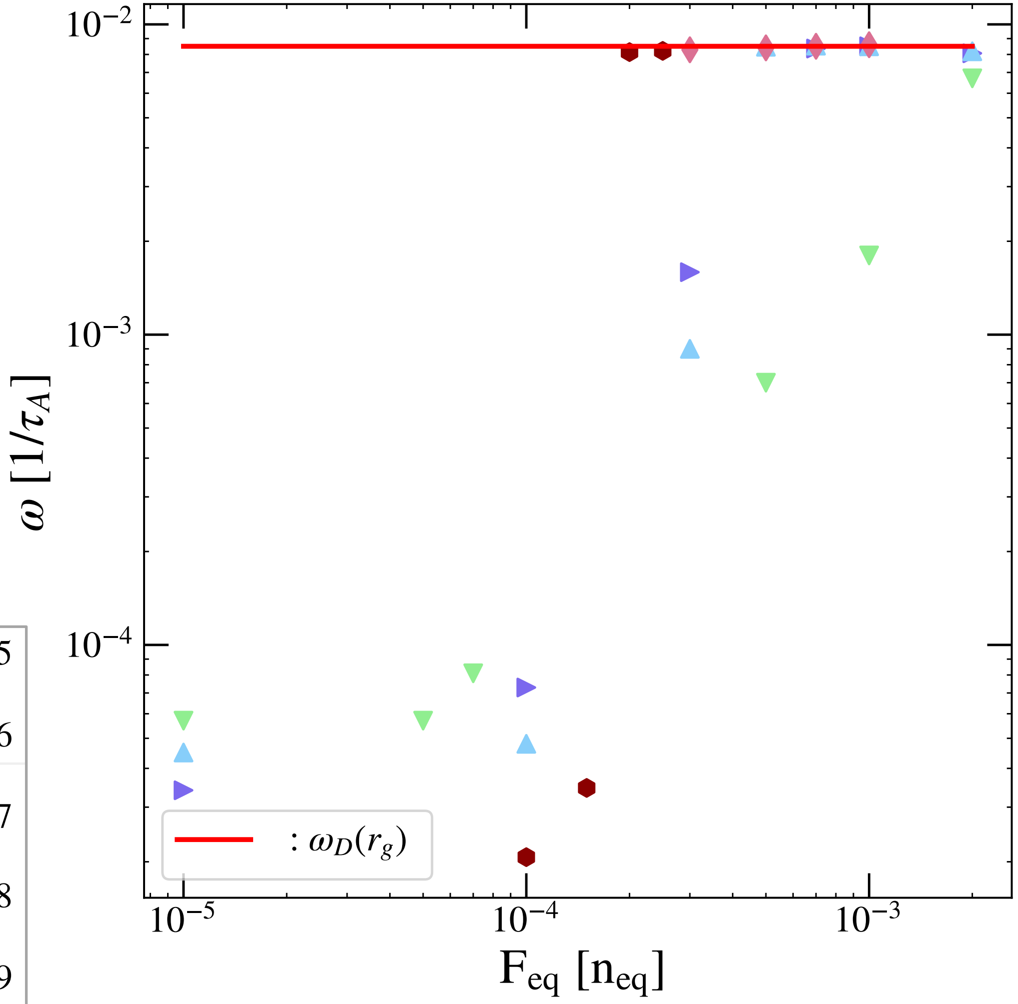
- Geometry :
  - Thermal Plasma : cylindrical 2D monohelicity ($m = n$)
  - Energetic Particles : toroidal φ, p_φ



- Coupling **MHD – kinetic** through $m, n = 1,1$

Linear Simulations





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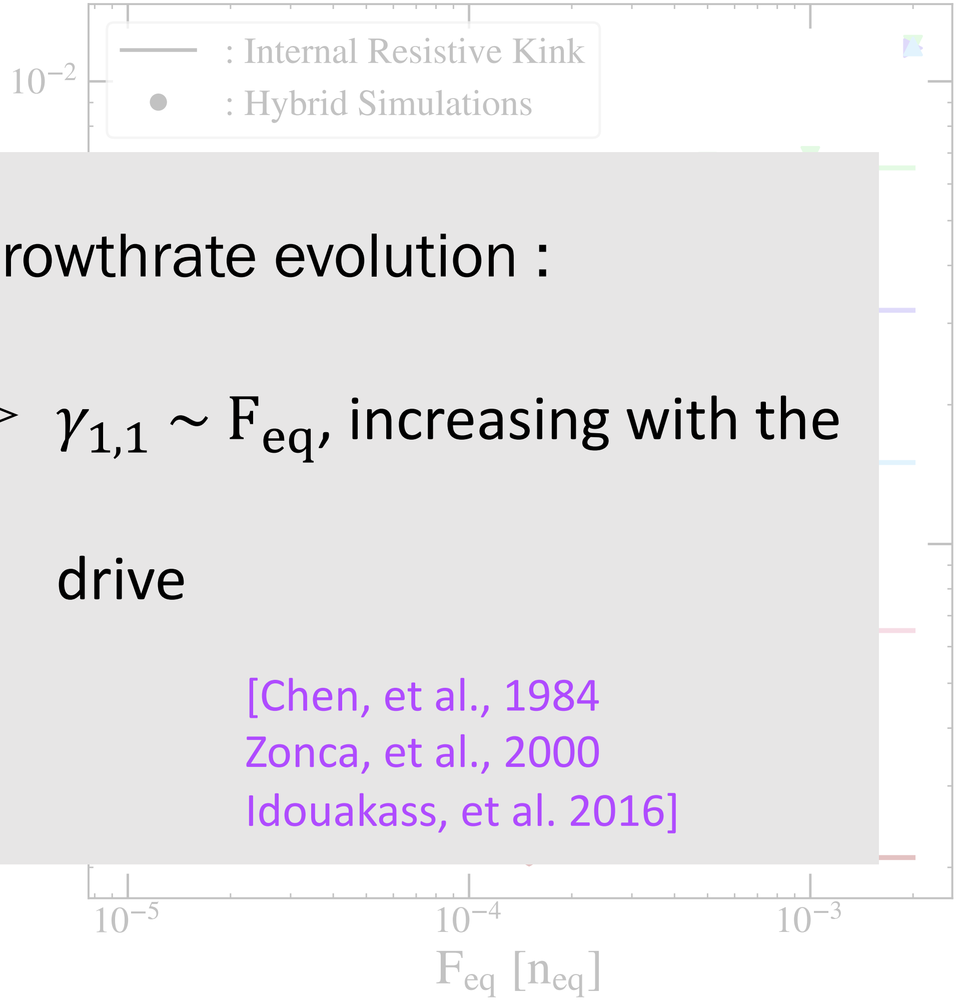
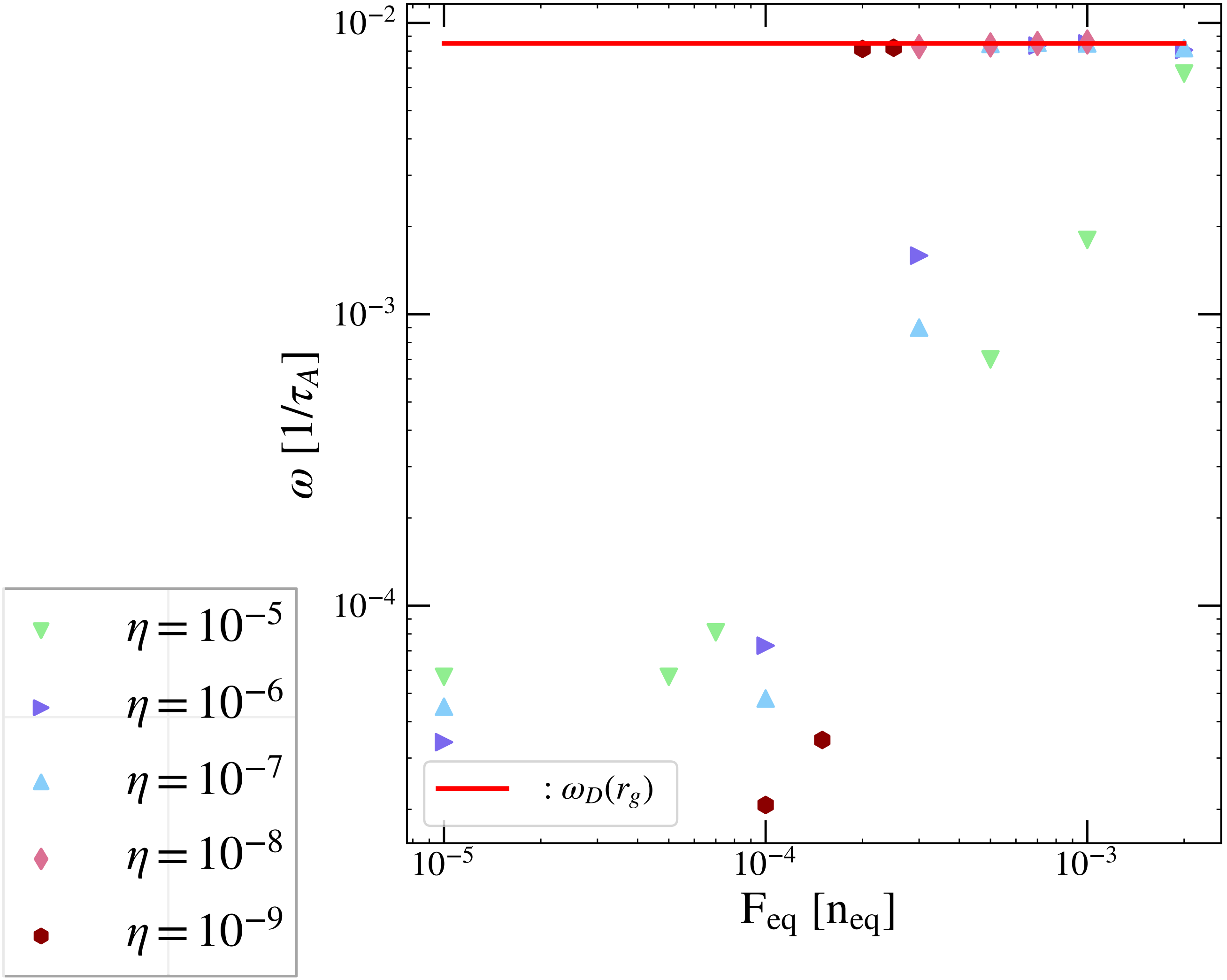


- Growthrate evolution :
  - $\gamma_{1,1} \sim \mathrm{F_{eq}}$, increasing with the drive

[Chen, et al., 1984
Zonca, et al., 2000
Idouakass, et al. 2016]



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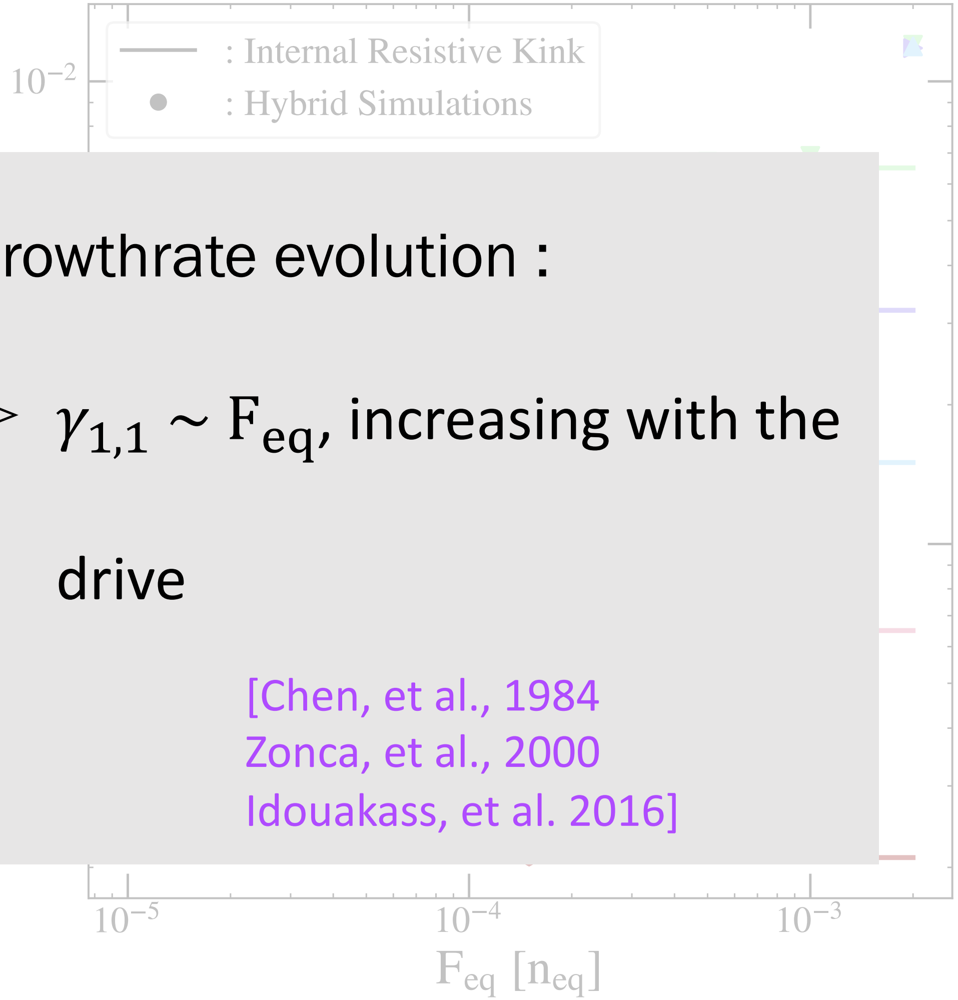

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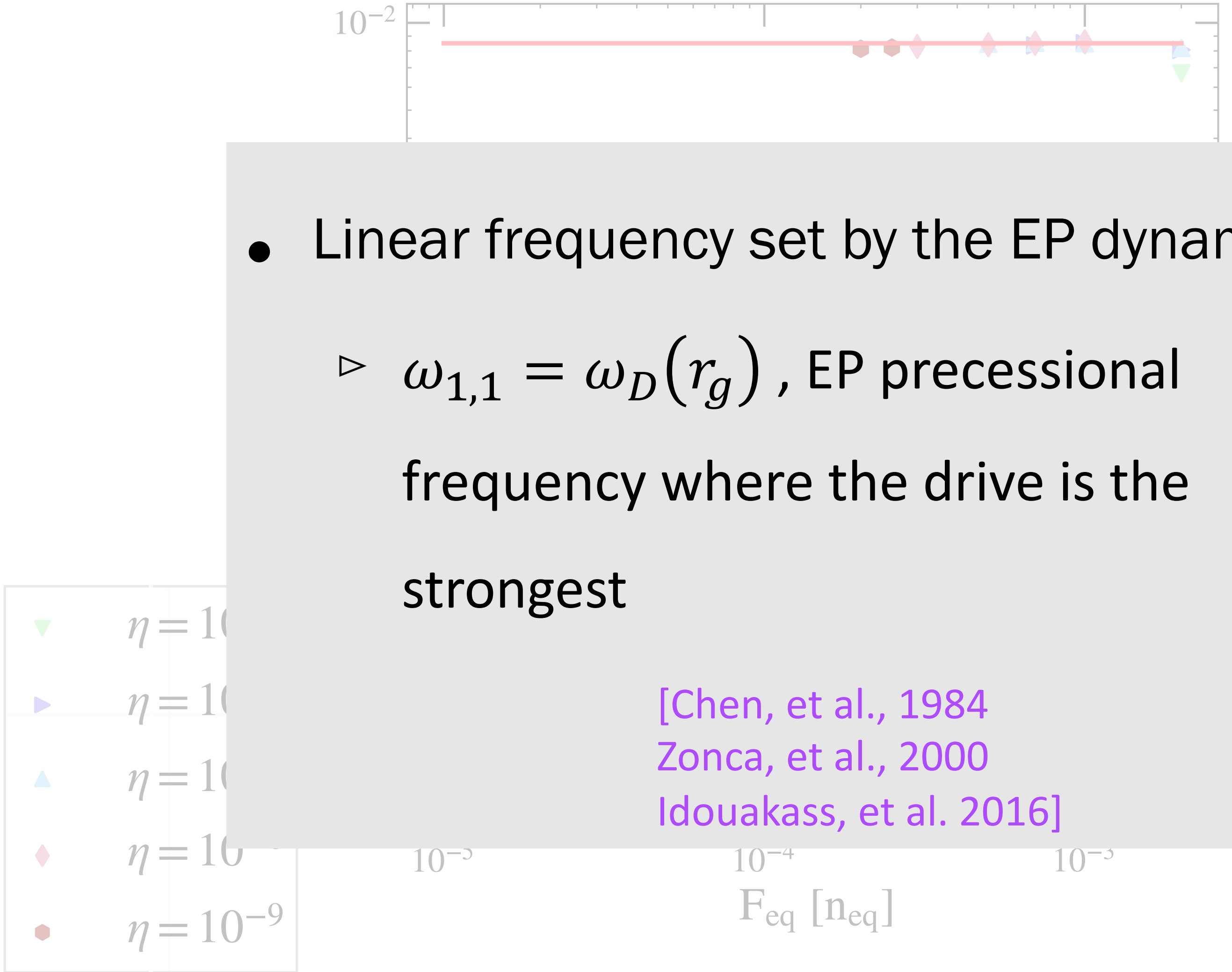
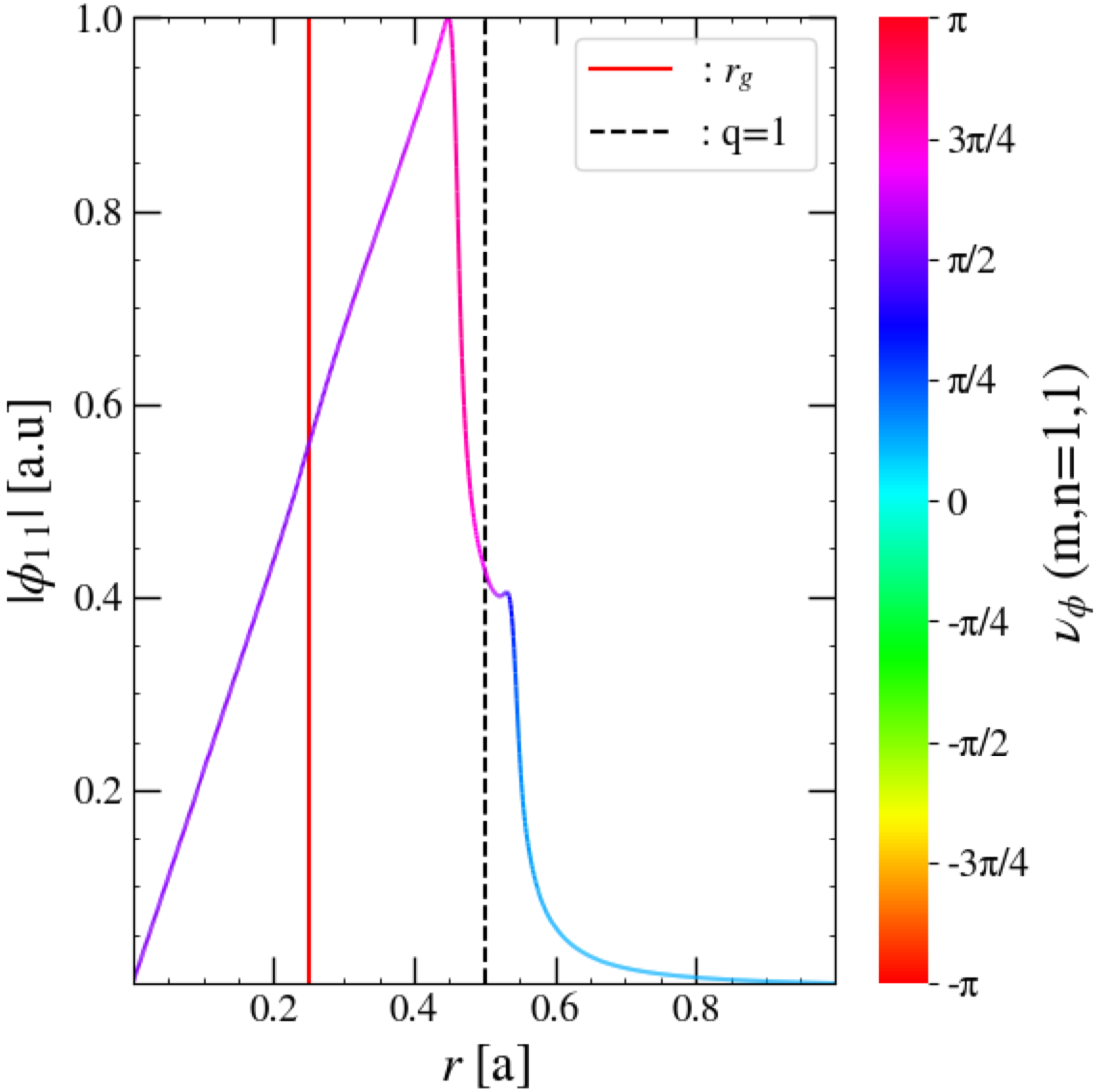


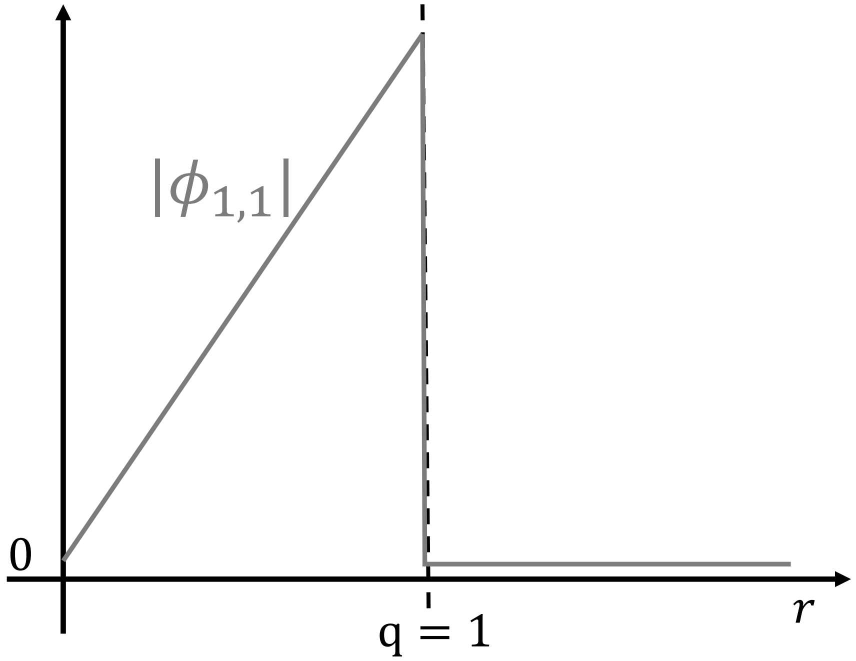
- Linear frequency set by the EP dynamic :
  - ▹ $\omega_{1,1} = \omega_D(r_g)$, EP precessional frequency where the drive is the strongest

[Chen, et al., 1984
Zonca, et al., 2000
Idouakass, et al. 2016]

Fishbone Eigenmode

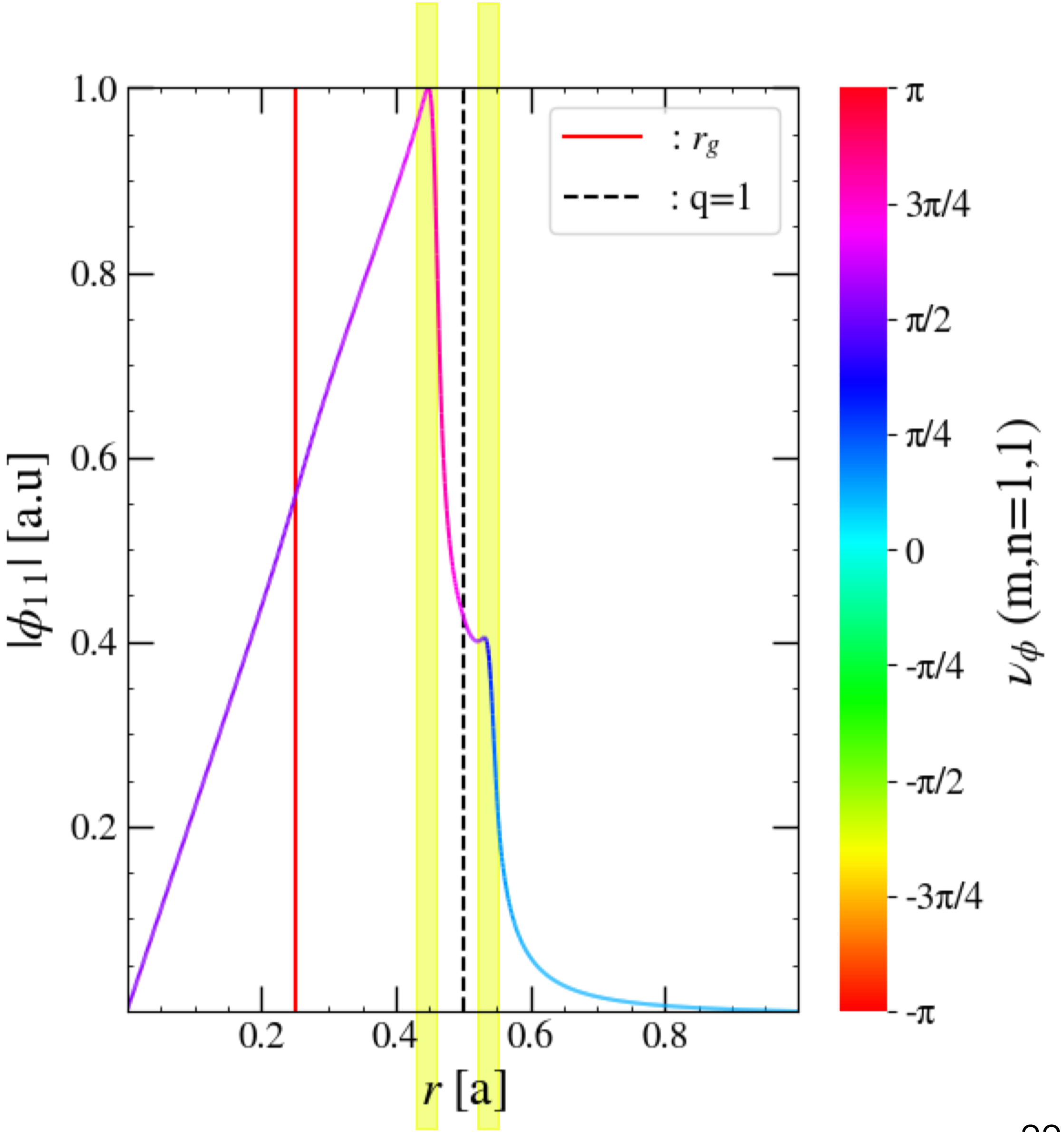


- Comparison with [Idouakass, et al. 2016] done in Slab geometry



Resistive Internal Kink Eigenfunctions without EP

Fishbone Eigenmode



- Comparison with [Idouakass, et al. 2016] done in Slab geometry

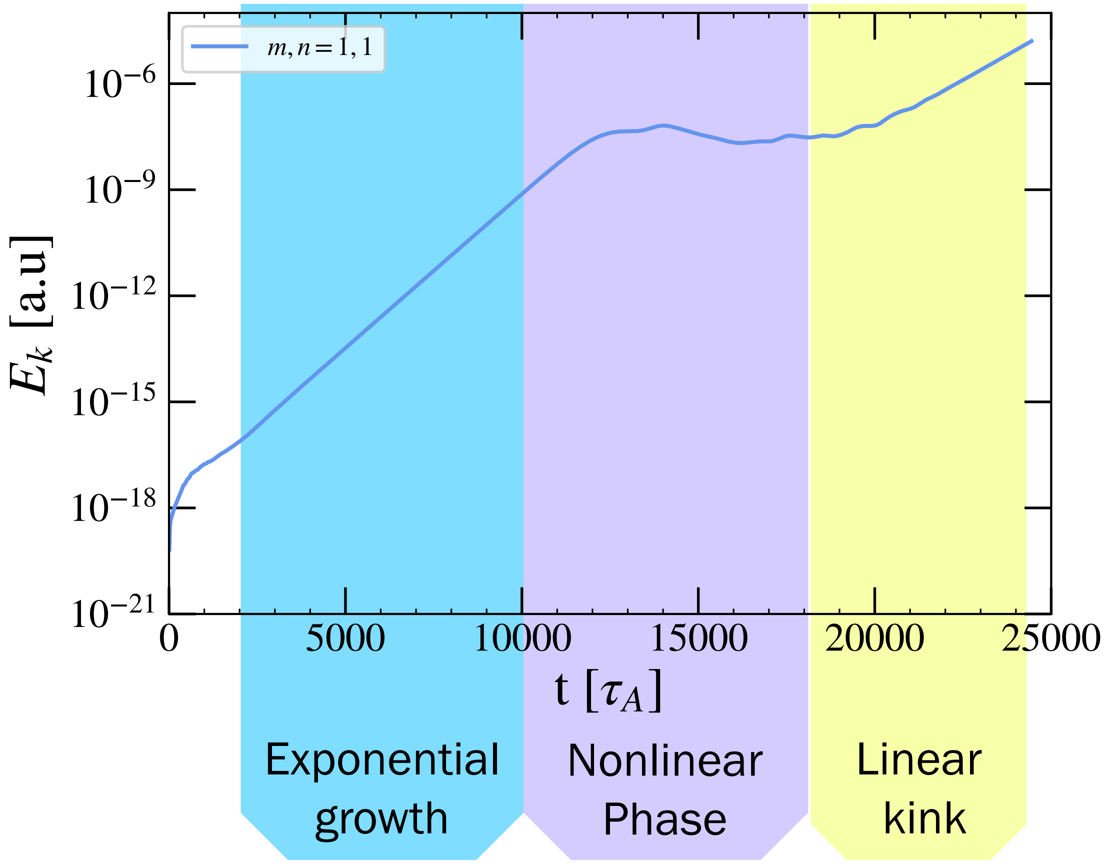
- Change of the kink mode shape without EP
  - ▹ Double peak around $q = 1$ surface at :

$$\omega_{1,1} = \pm k_\parallel V_A(r)$$

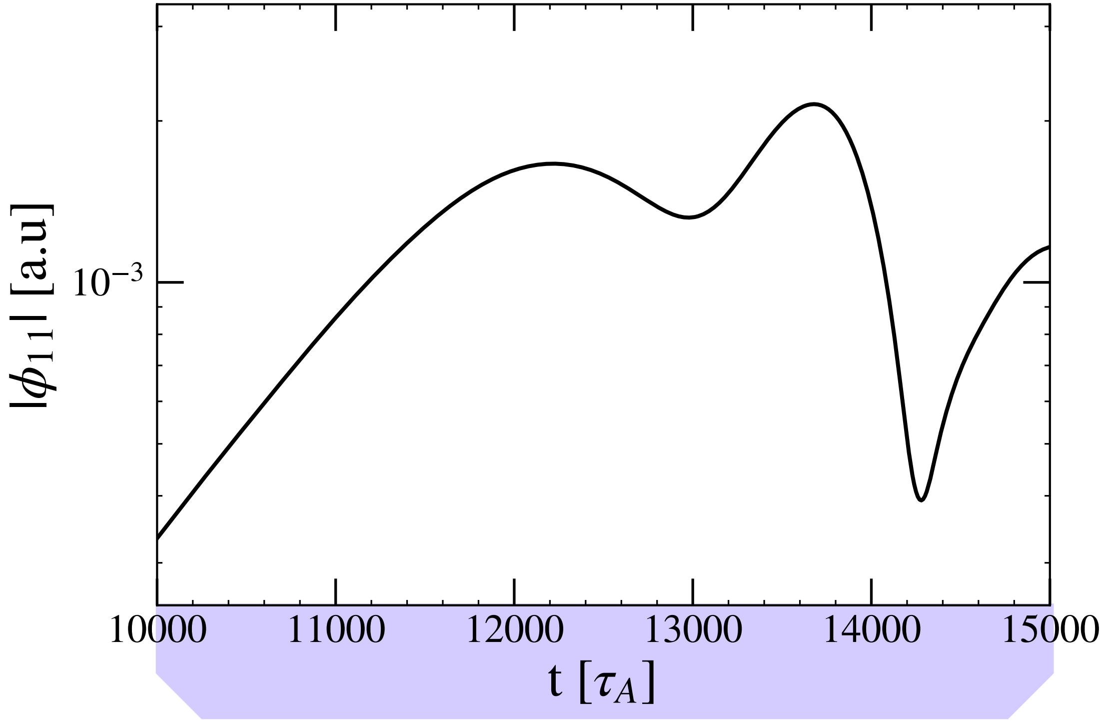
Resistive Internal Kink Eigenfunctions without EP

Nonlinear Fishbone (close to the threshold, MHD nonlinear effects : OFF)

- Bulk plasma kinetic energy



- Amplitude of the fishbone mode $\phi_{1,1}$

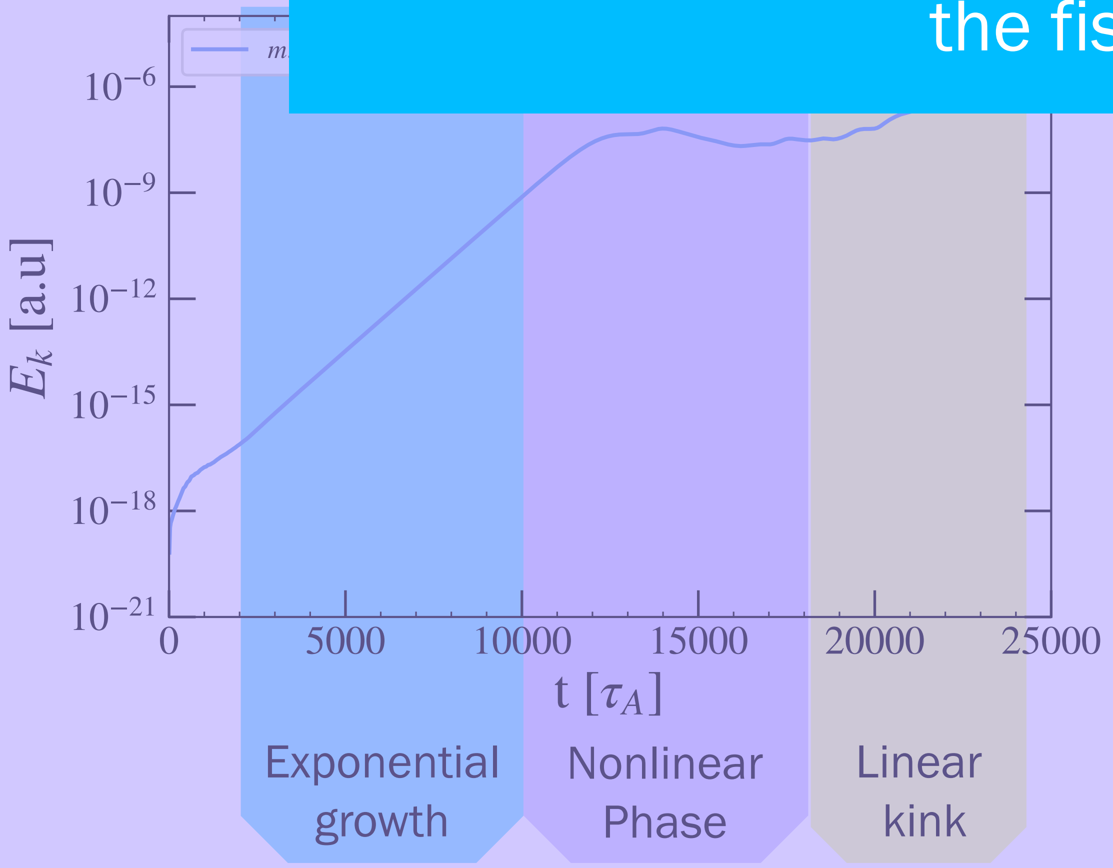


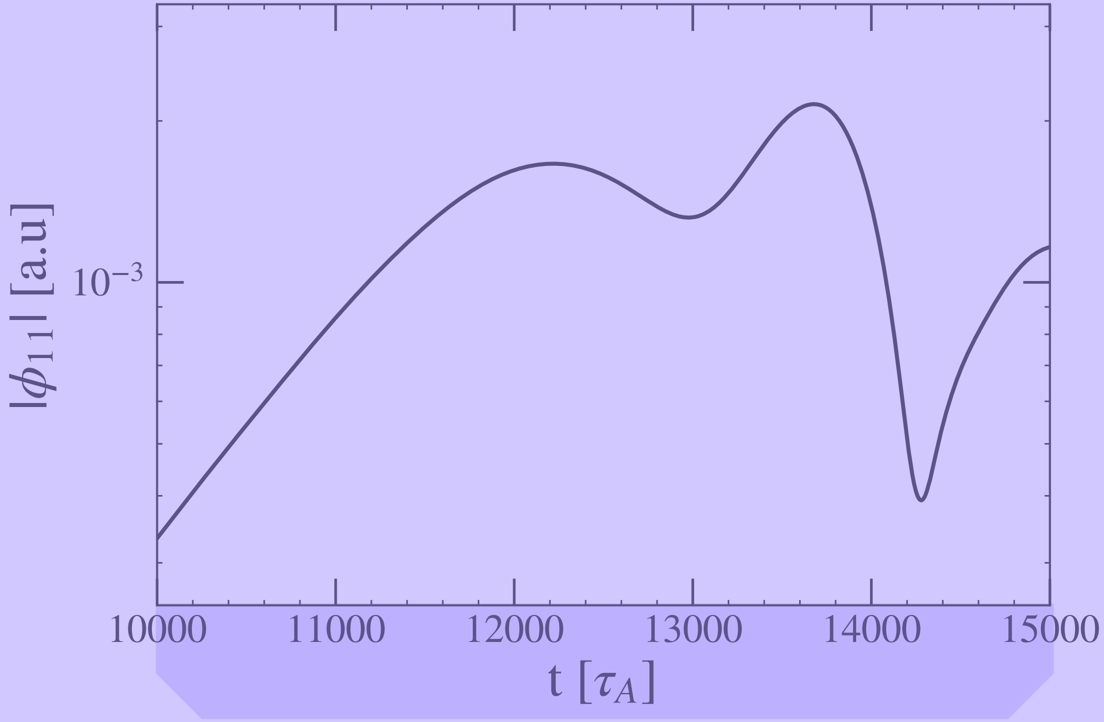
Nonlinear Fishbone (close to the threshold, MHD nonlinear effects : OFF)

- Bulk pla

What nonlinear process is responsible for the saturation of the fishbone mode?

e $\phi_{1,1}$





Nonlinear Fishbone (close to the threshold, MHD nonlinear effects : OFF)

- Bulk pla... $\phi_{1,1}$

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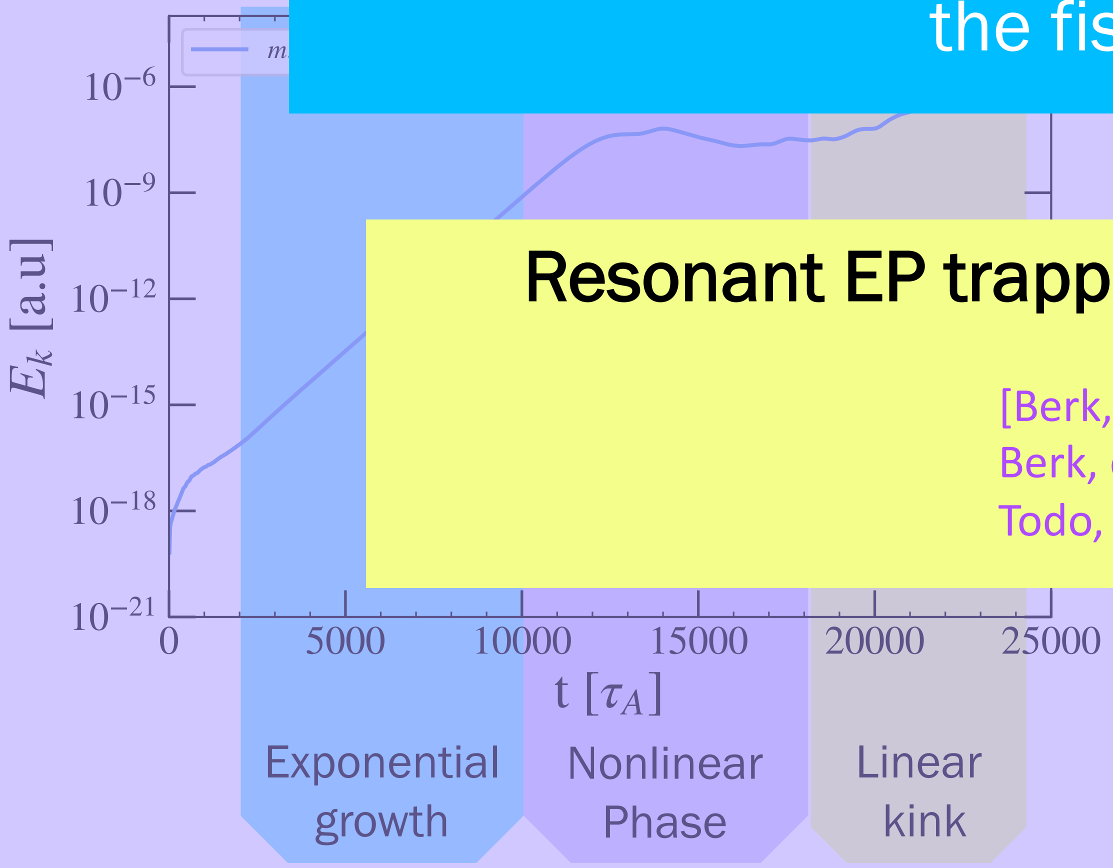
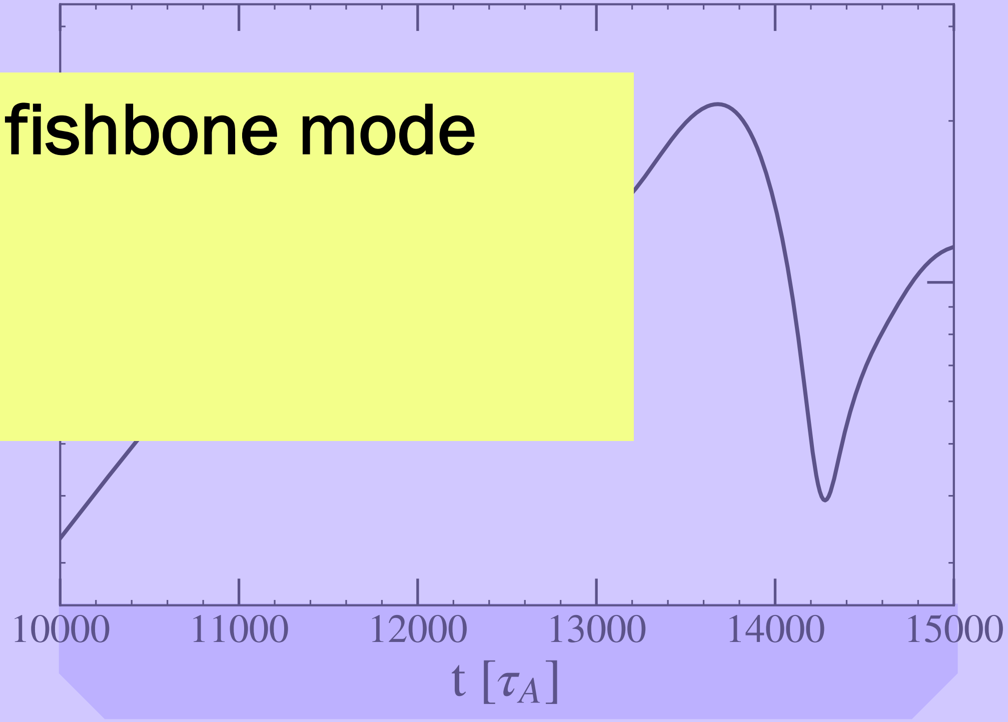




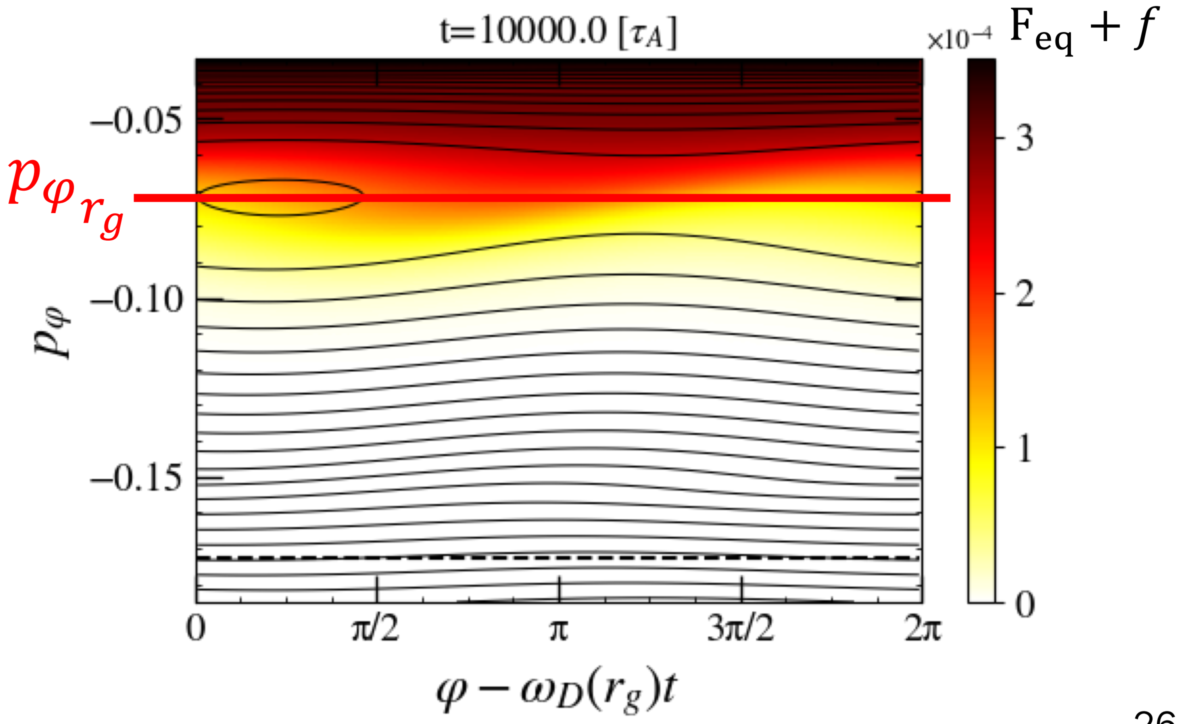
Resonant EP trapped by the fishbone mode

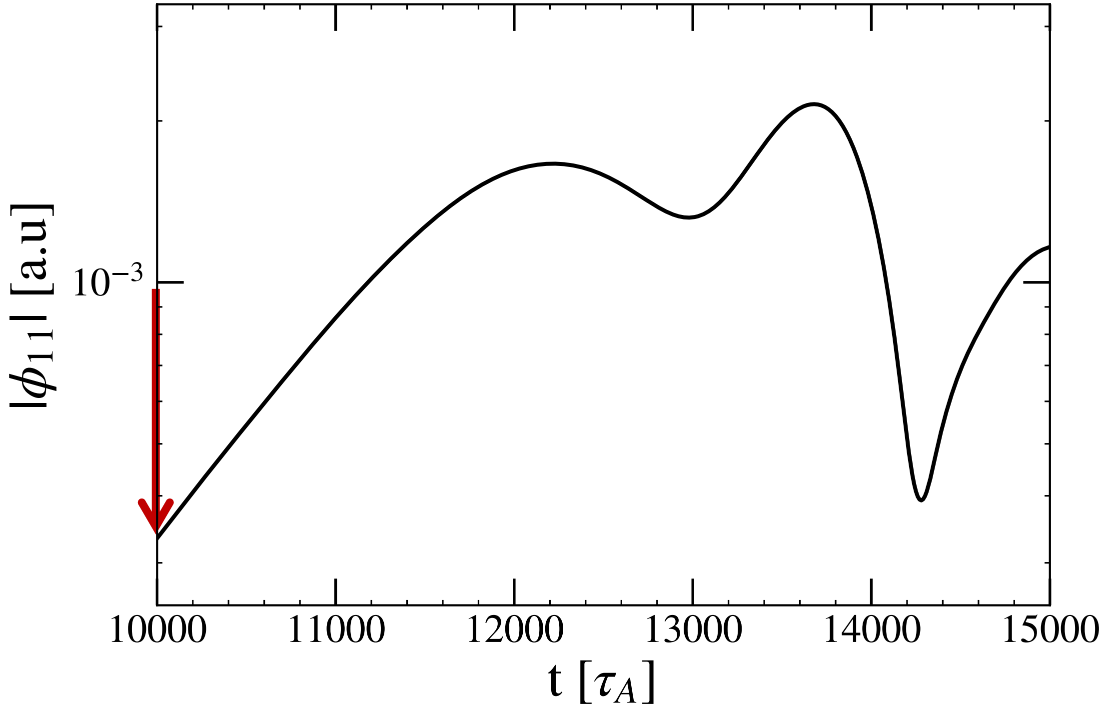
[Berk, et al., 1990
Berk, et al., 1992,
Todo, et al.,1995]

Energy Exchange between mode and resonant EP

(close to the threshold, MHD nonlinear effects : OFF)

- EP distribution function in the resonant EP :
  - Frame moving at $\omega_D(r_g)$ in the direction φ
  - Isoline of $\mathrm{H_{eq}} + H - \omega_D(r_g)p_\varphi$





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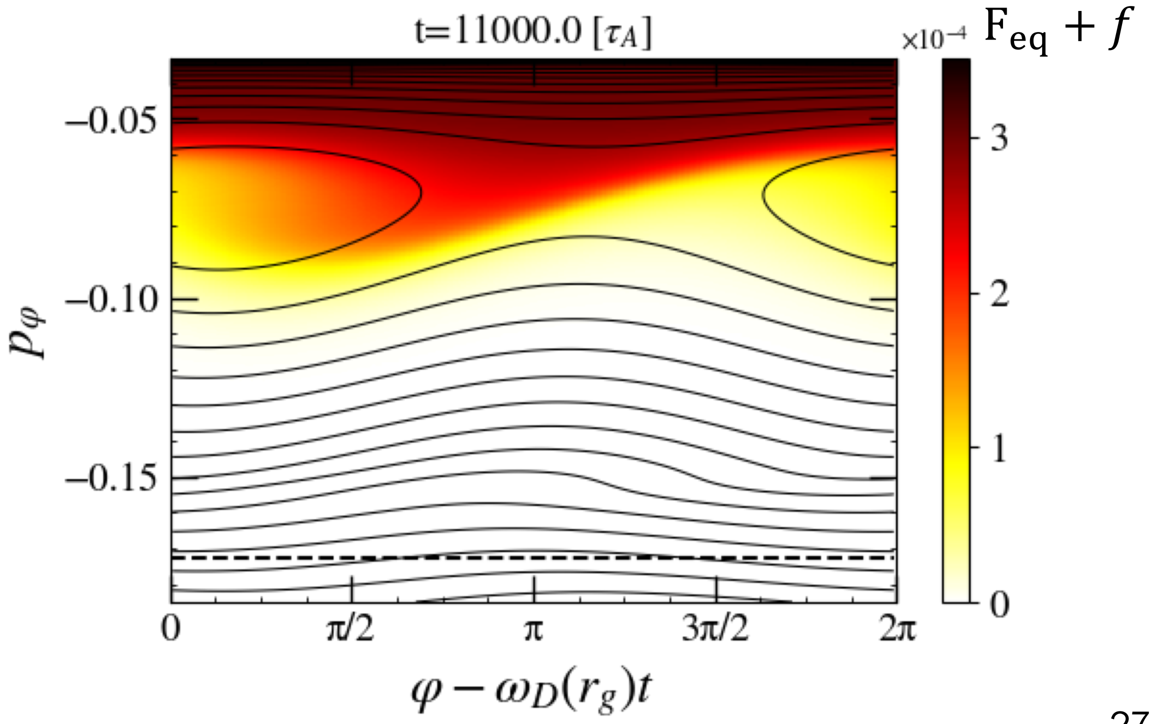

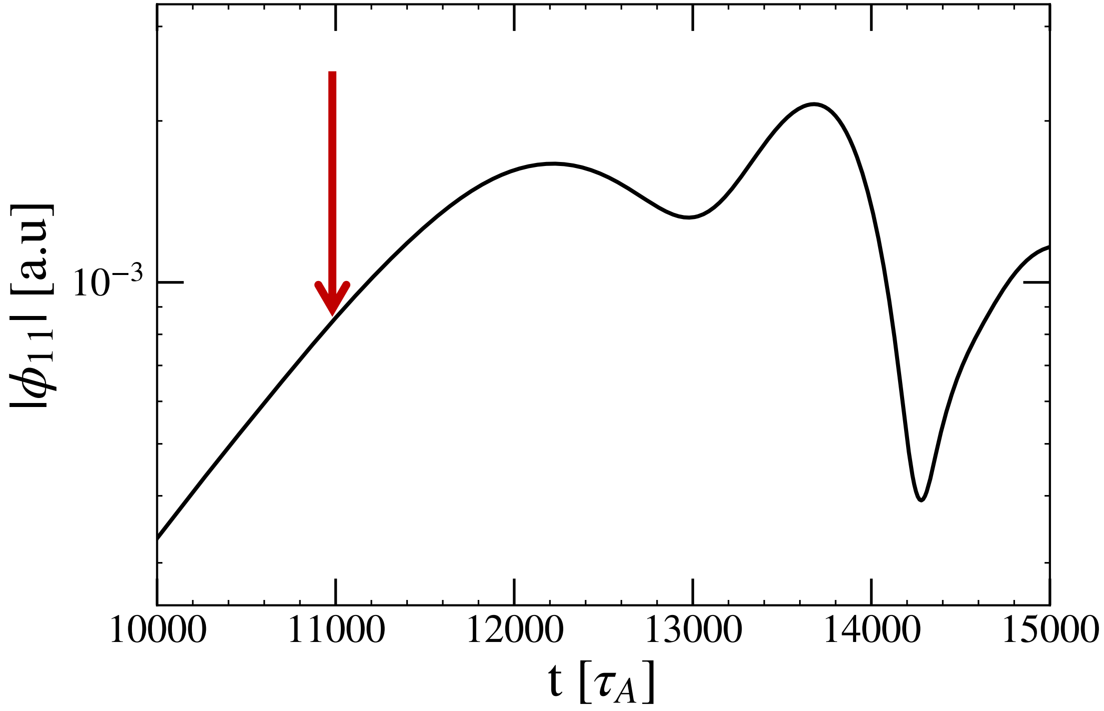

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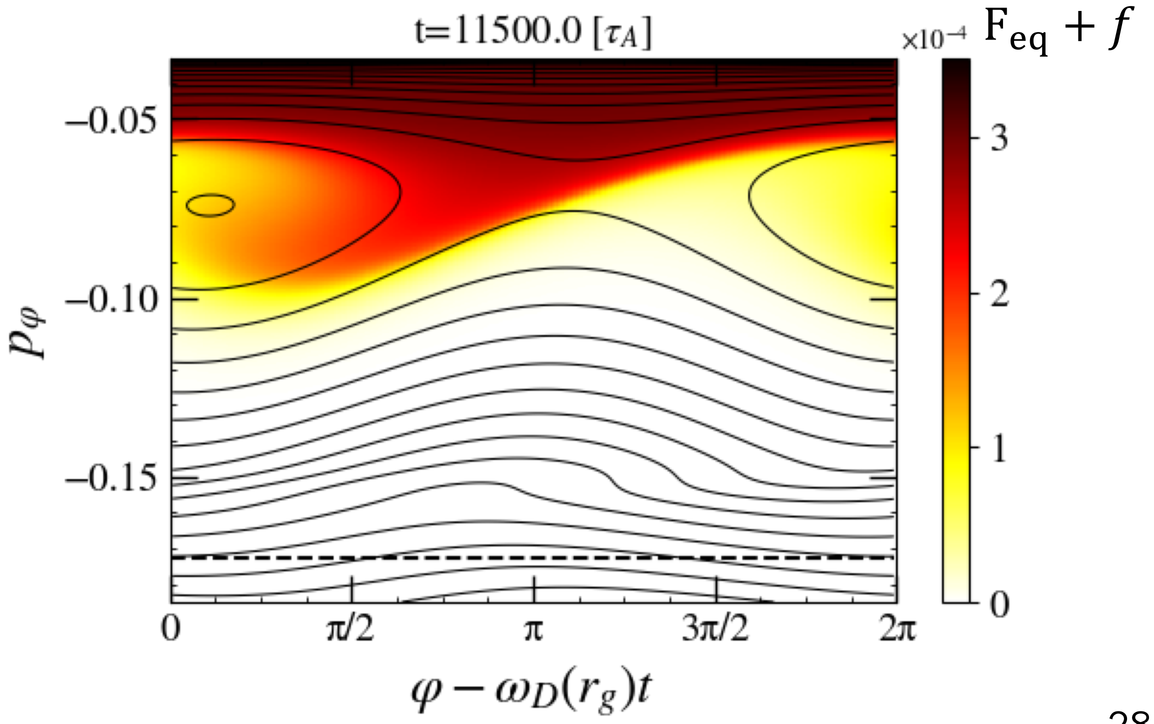

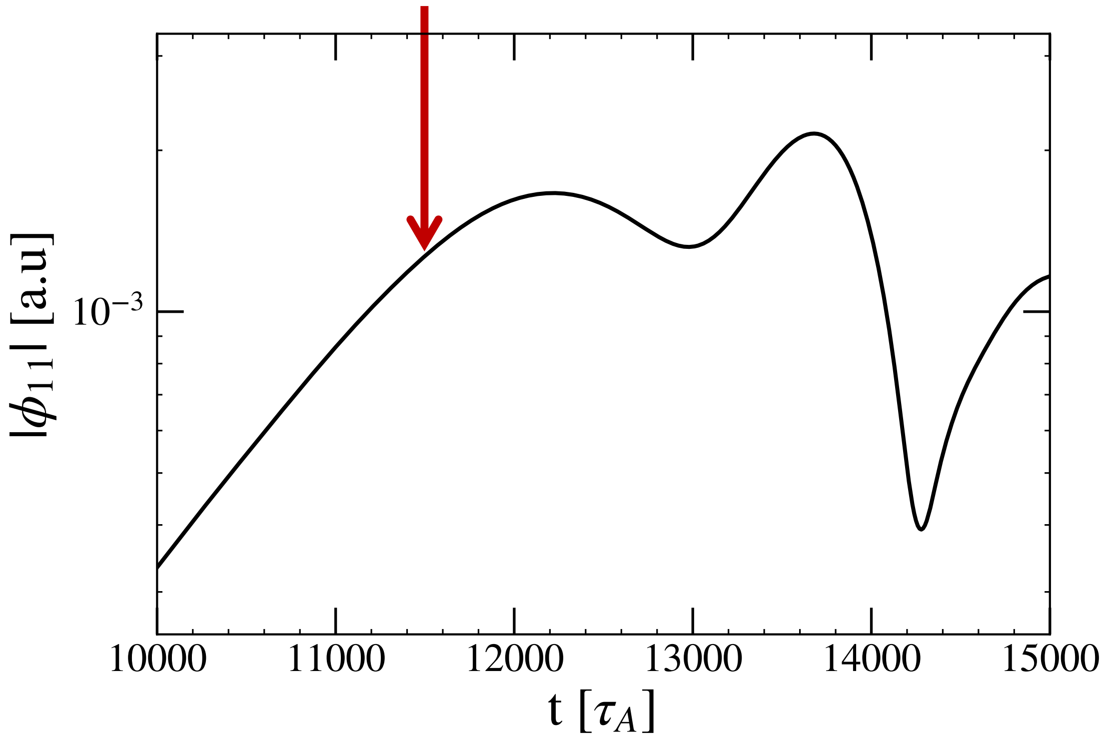

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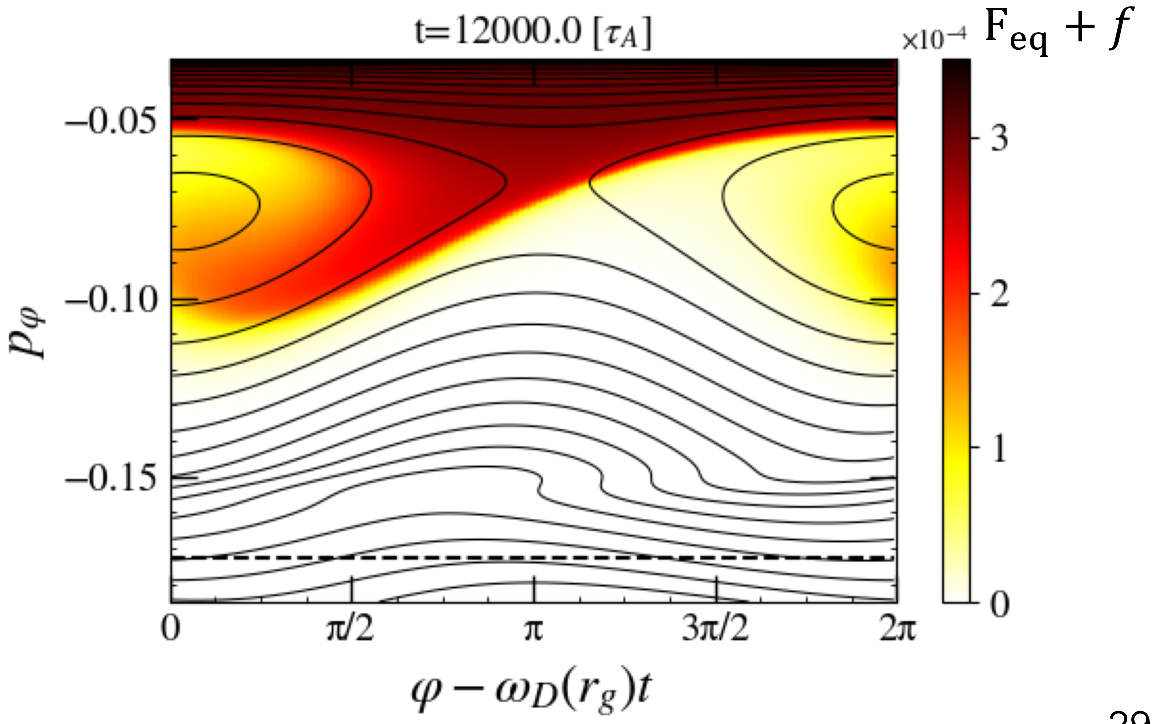

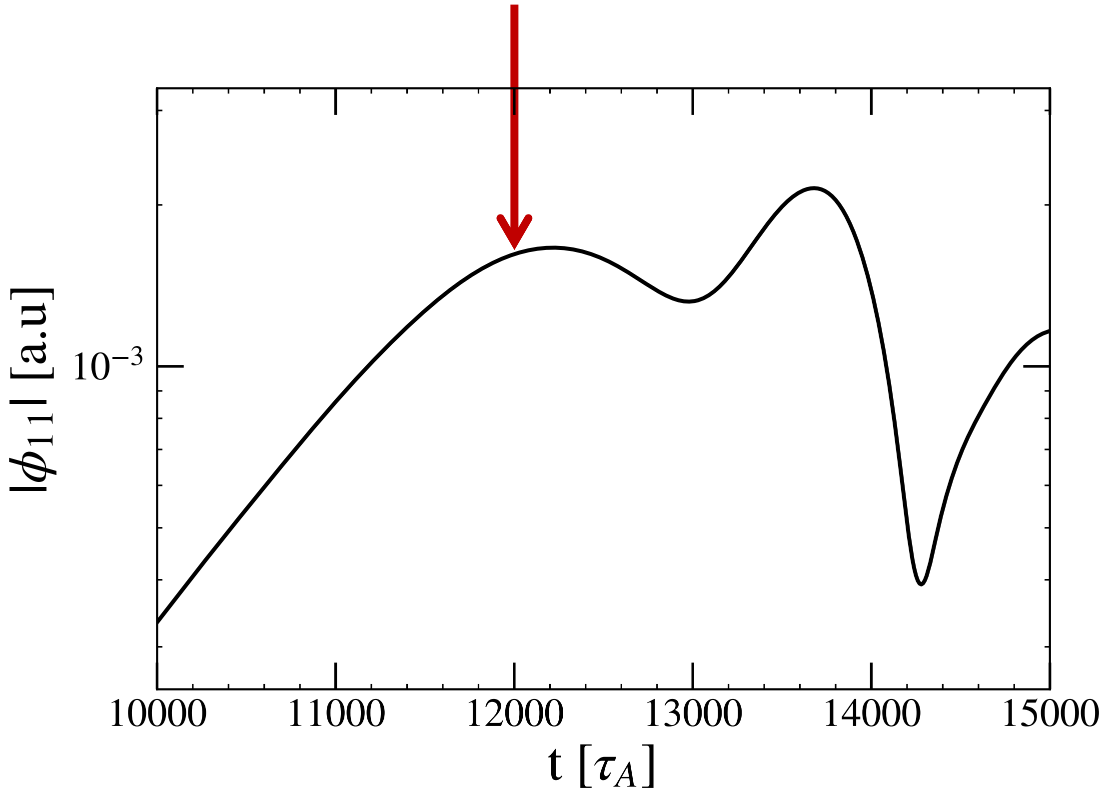

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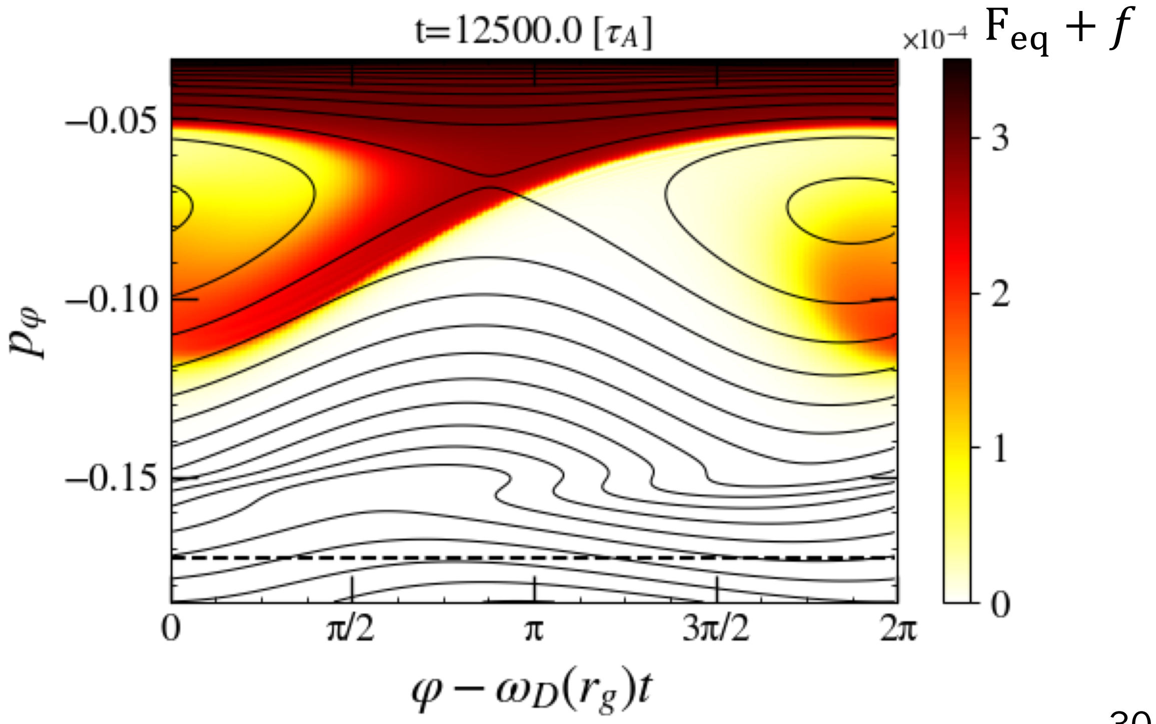
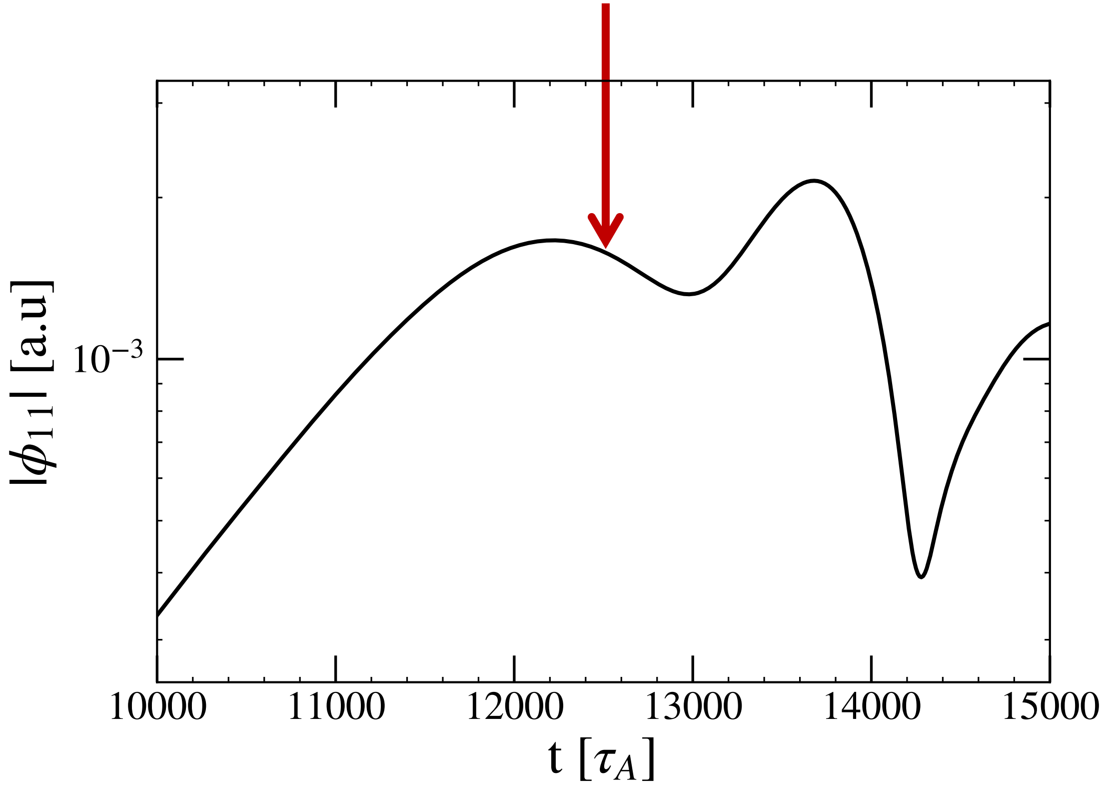
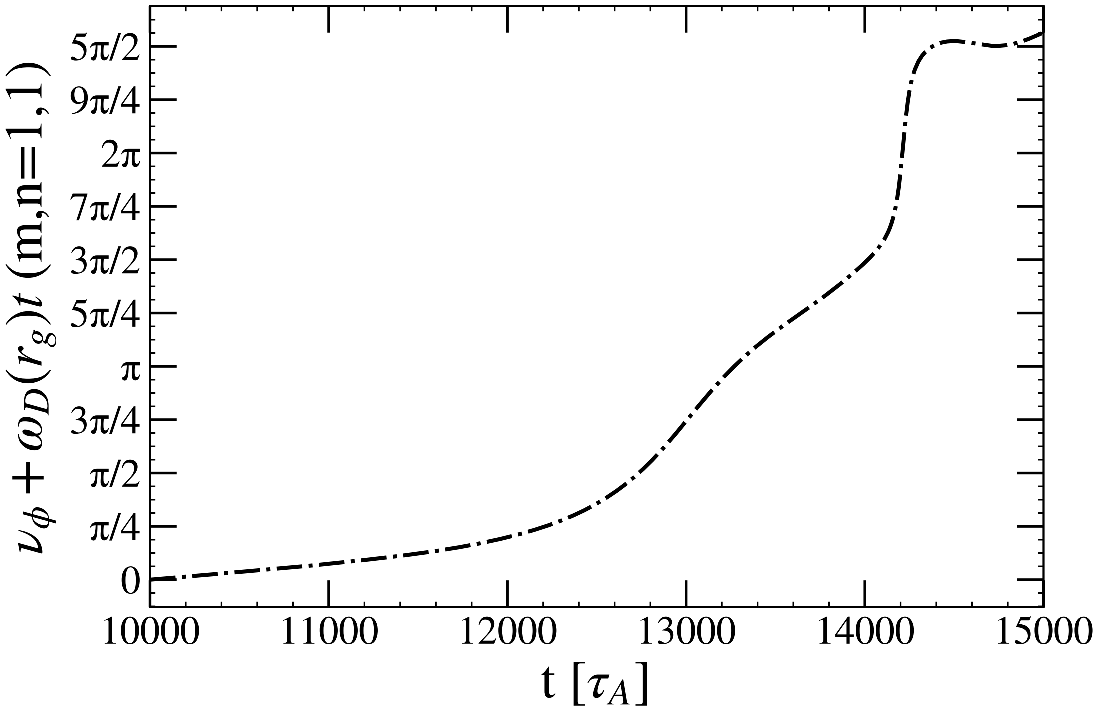




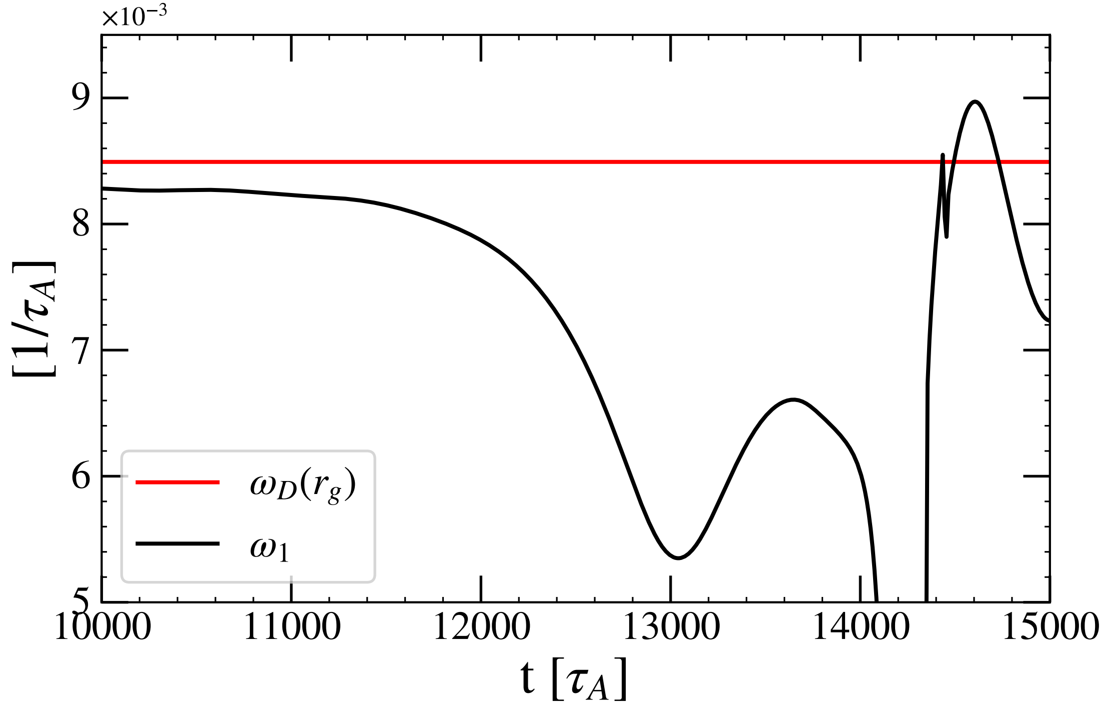
Resonance Condition Lost

(close to the threshold, MHD nonlinear effects : OFF)

- Phase evolution of $\phi_{1,1}$:
  - Frame of the resonant interaction



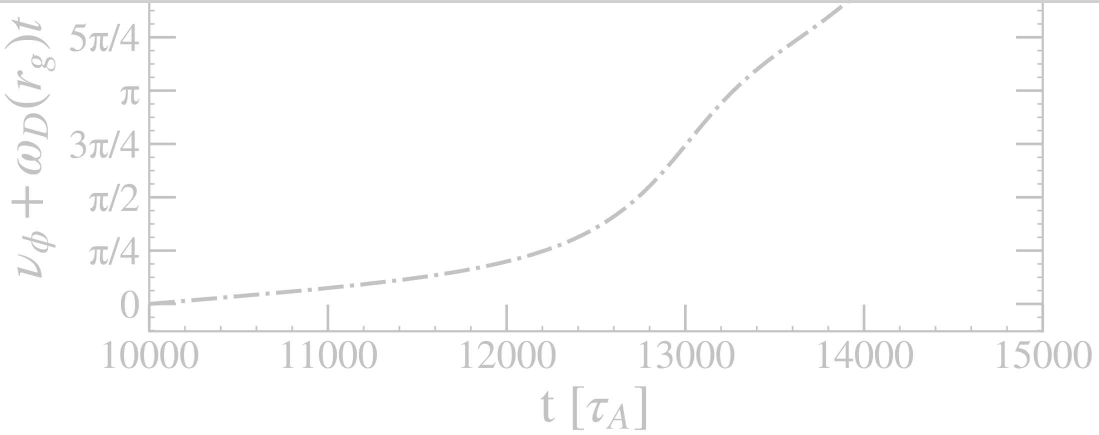
- Evolution of the frequency of $\phi_{1,1}$:



Resonance Condition Lost

(close to the threshold, MHD nonlinear effects : OFF)

- Lost of resonance between EP and $\phi_{1,1}$:
  - ▹ $f_{1,1}$ changing their frequency for $\omega_D(p_\varphi)$
  - ▹ $f_{1,1}$ frequency decreases as $\omega_D \propto p_\varphi$



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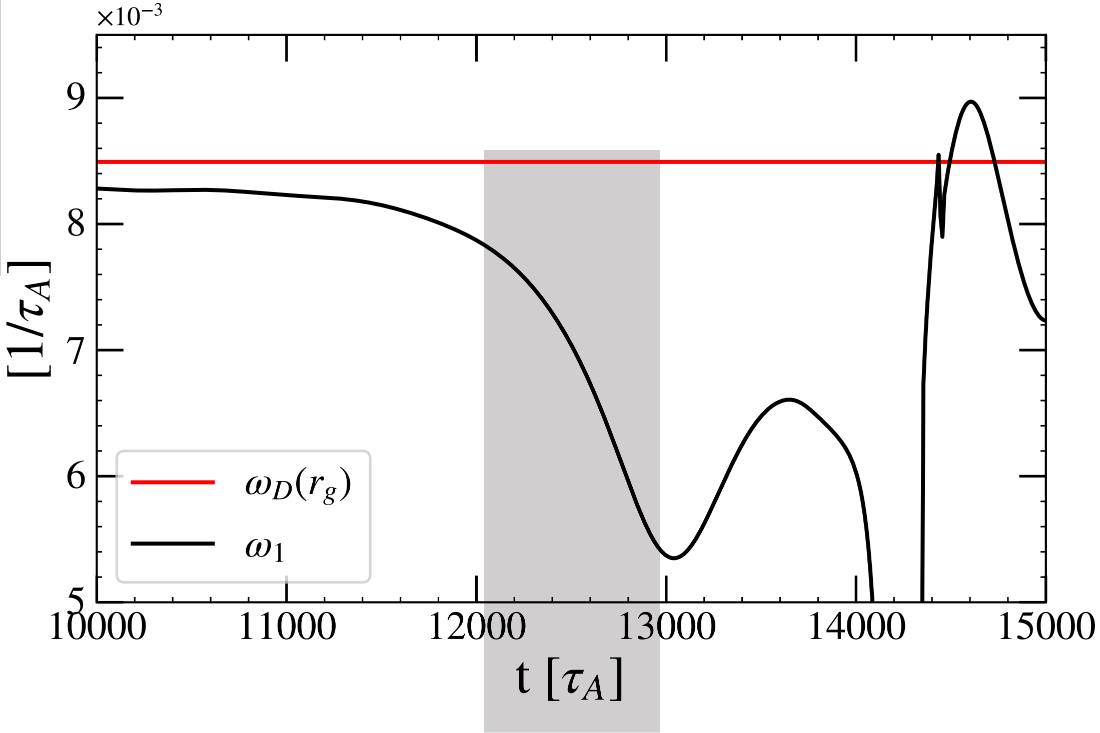

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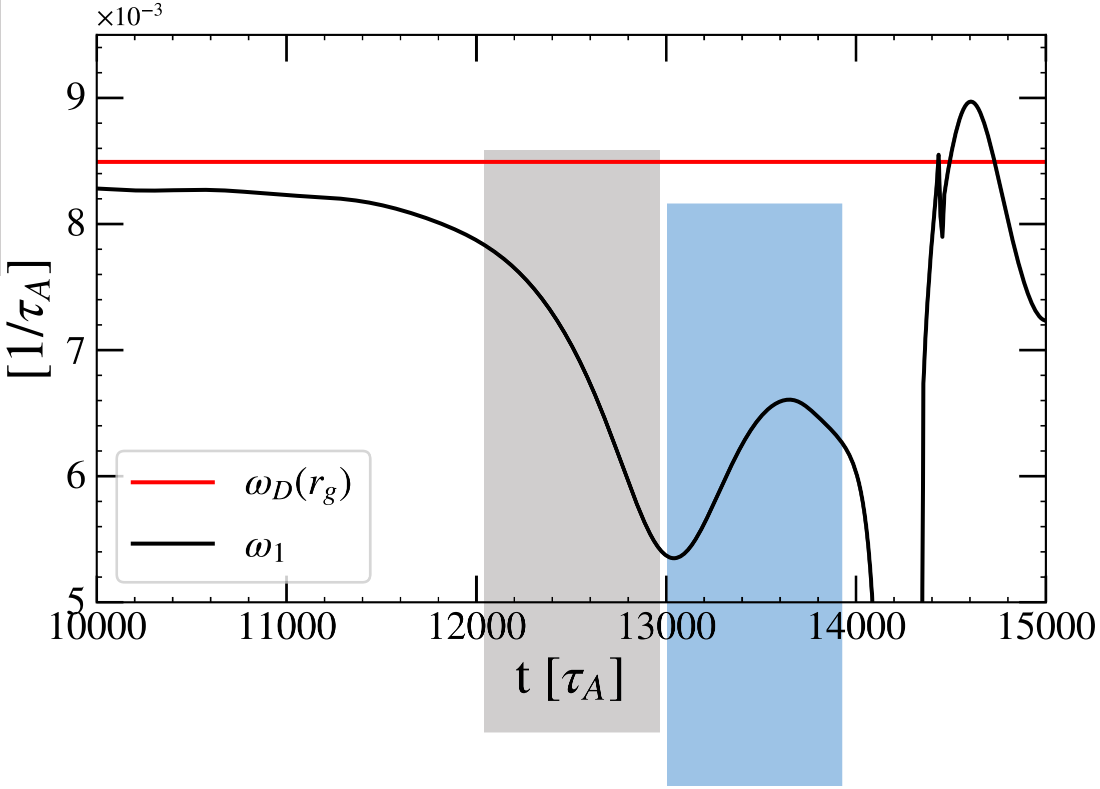
  - ▹ $f_{1,1}$ frequency decreases as $\omega_D \propto p_\varphi$

- New synchronization between EP and $\phi_{1,1}$:
  - ▹ New trapping of the resonant EP

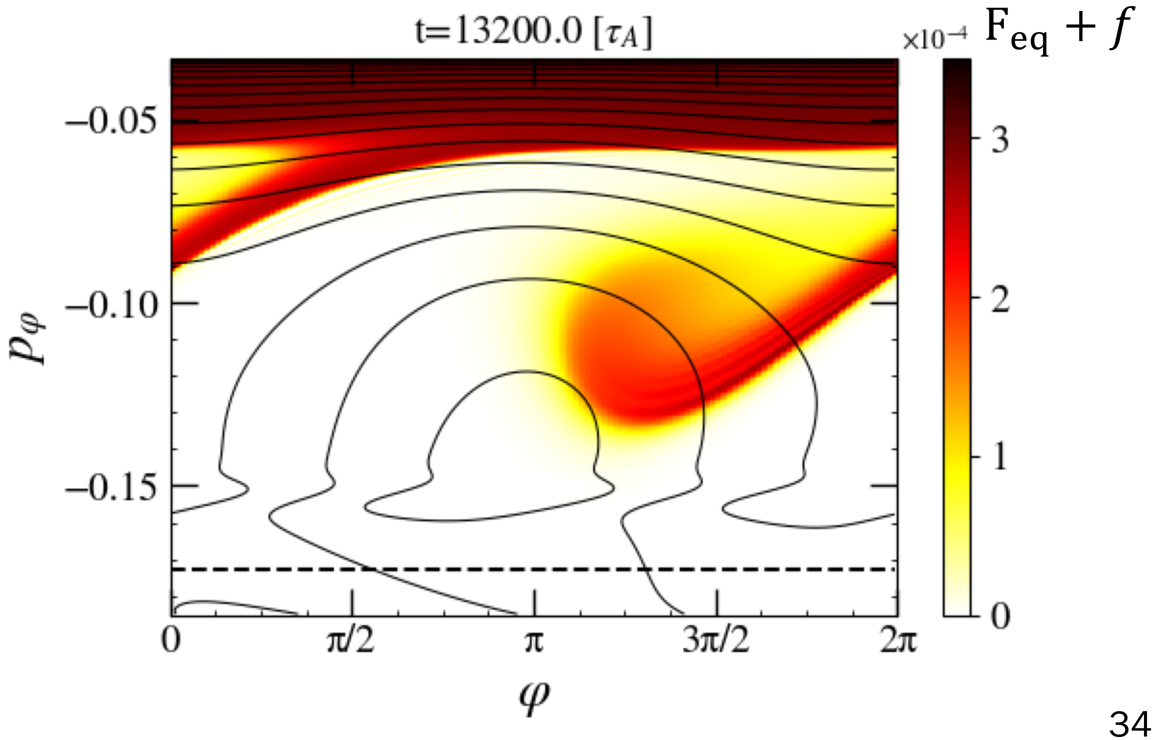
- Evolution of the frequency of $\phi_{1,1}$:

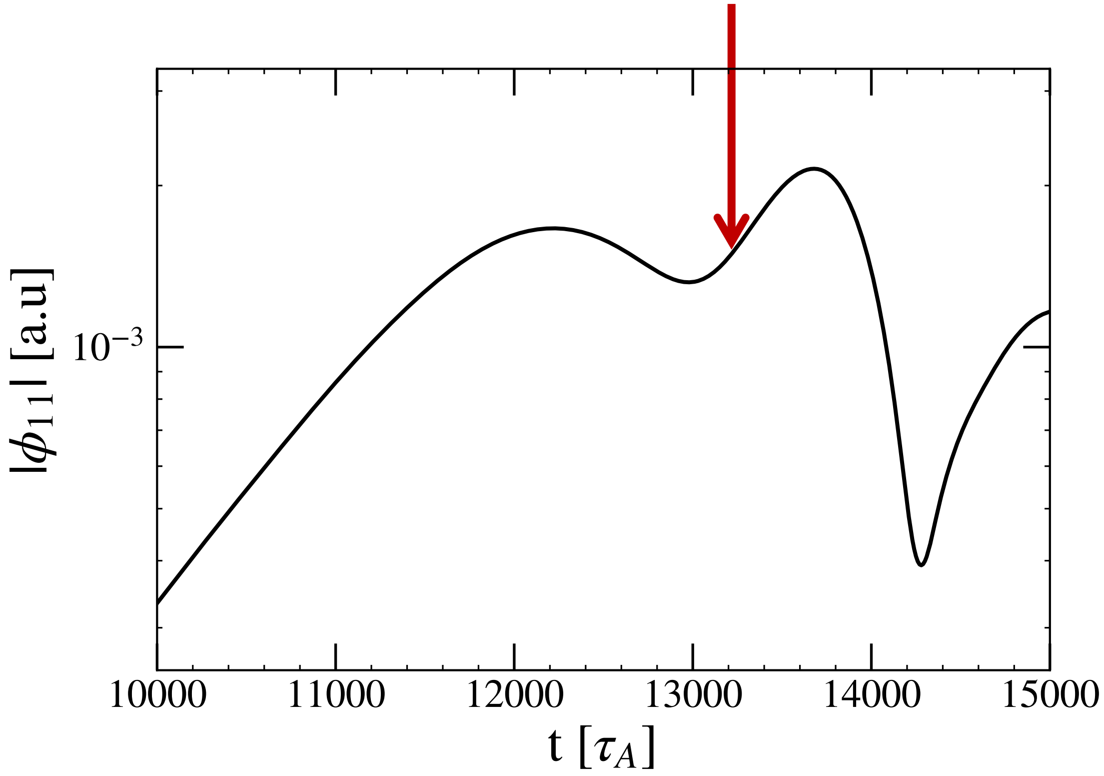


Energy Exchange between mode and resonant EP

(close to the threshold, MHD nonlinear effects : OFF)

- EP distribution function in the resonant EP :
  - Isoline of $\mathrm{H_{eq}} + H - \omega_{\phi_{1,1}}(t)p_\varphi$





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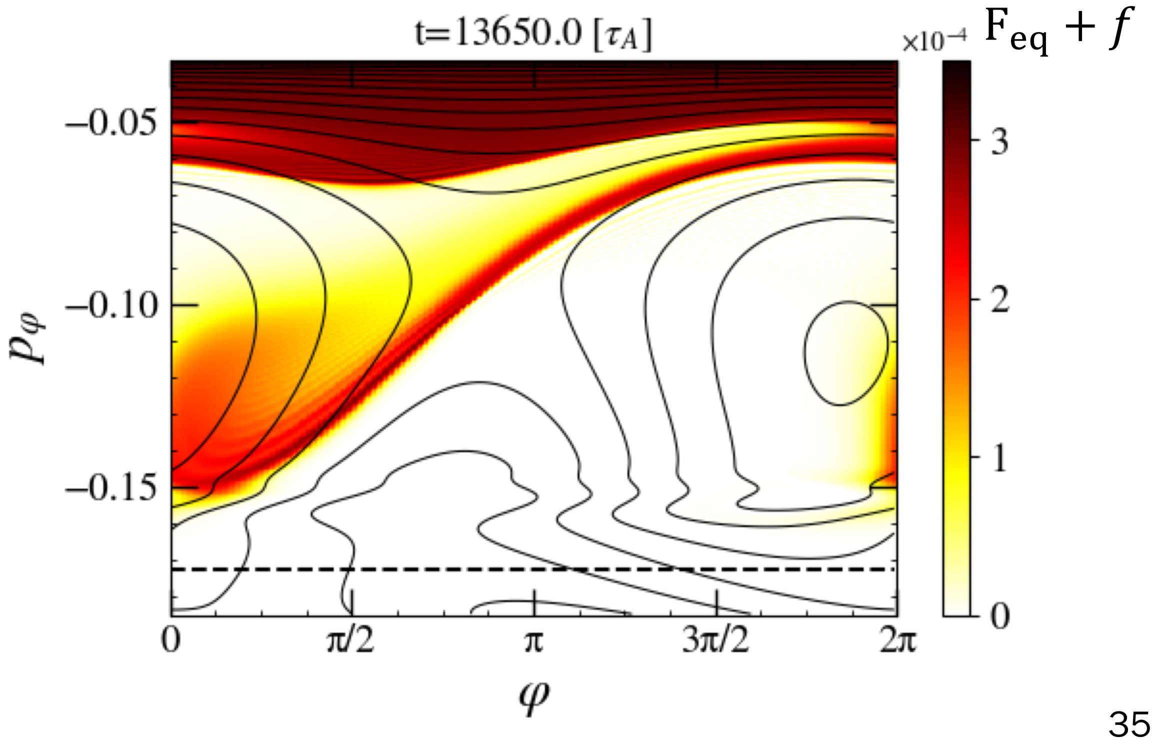

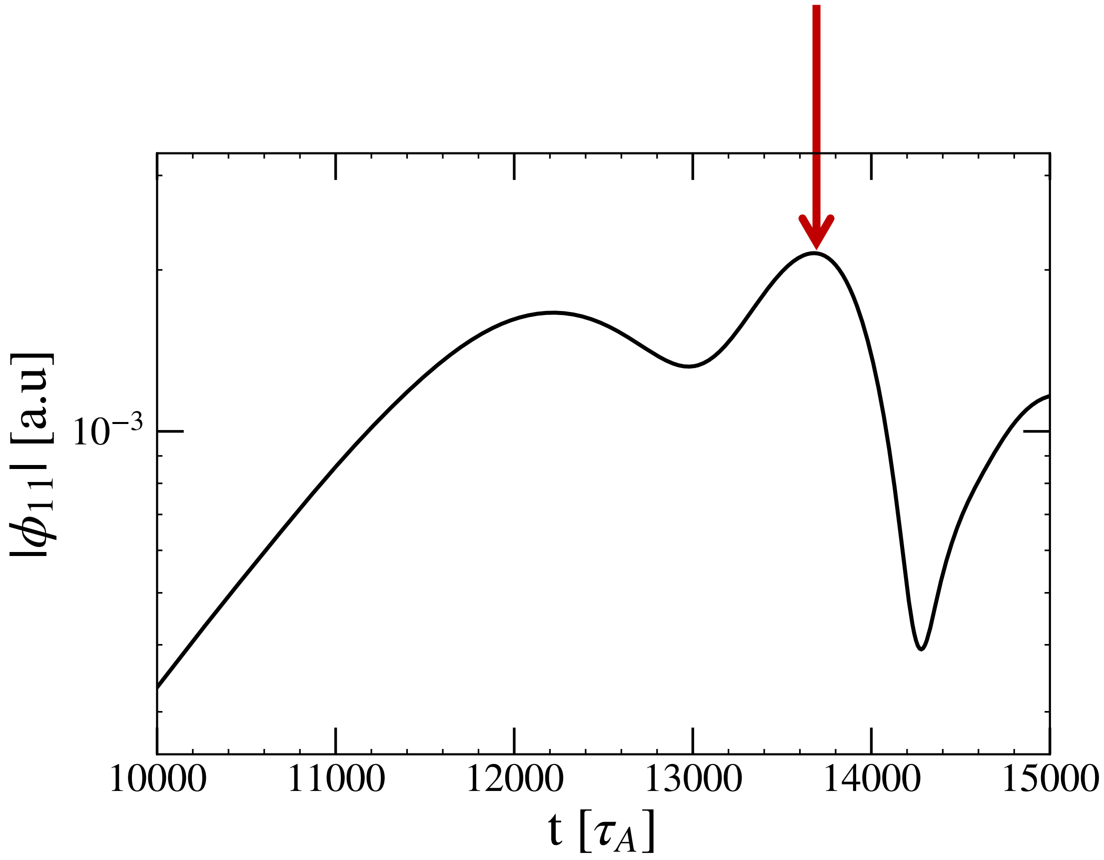

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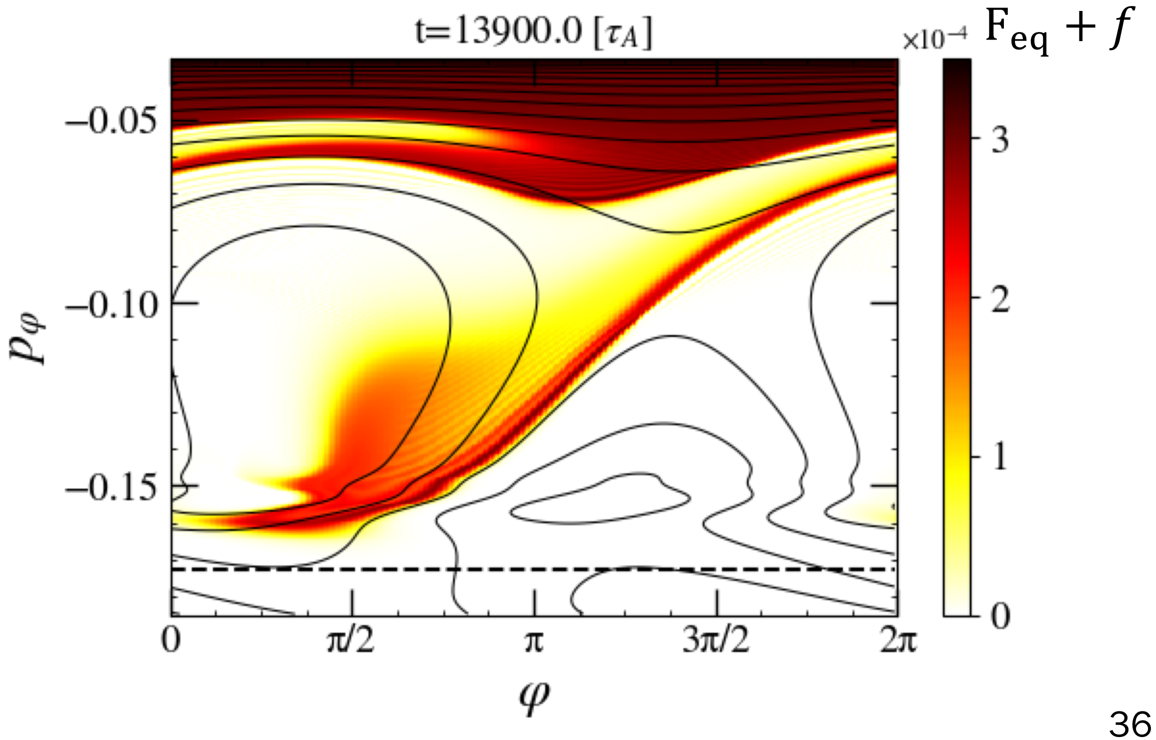
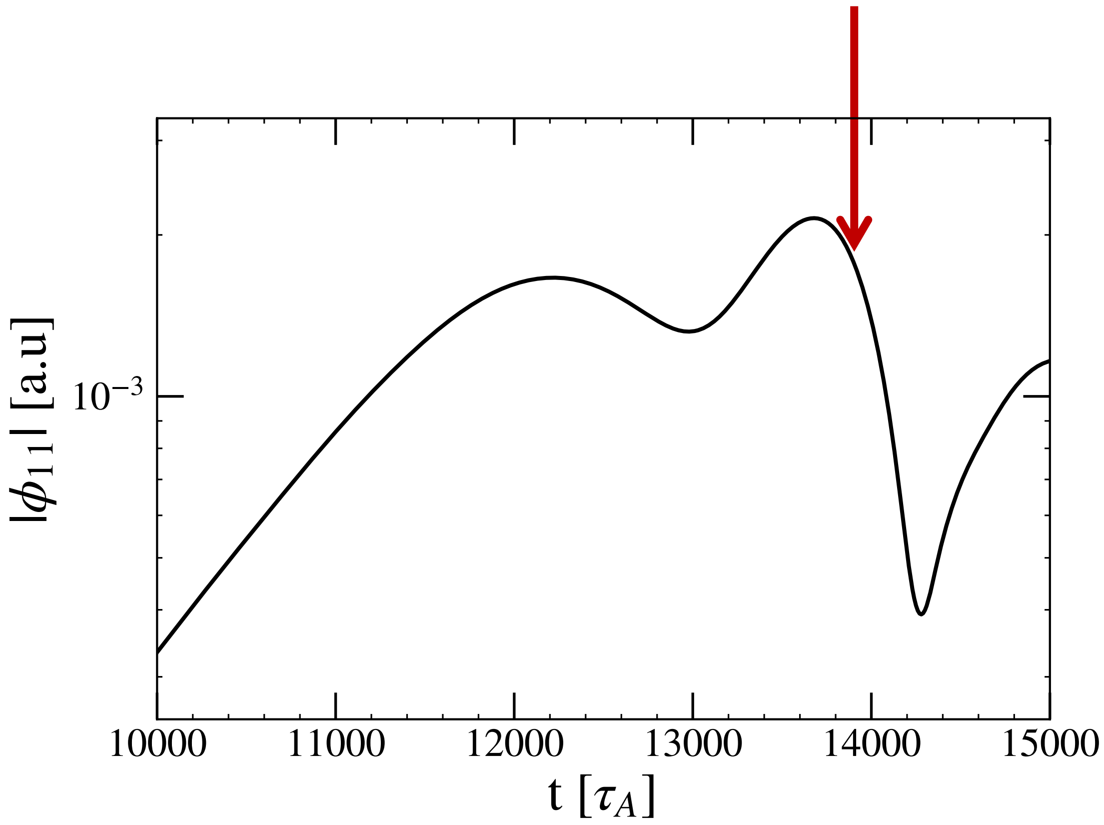
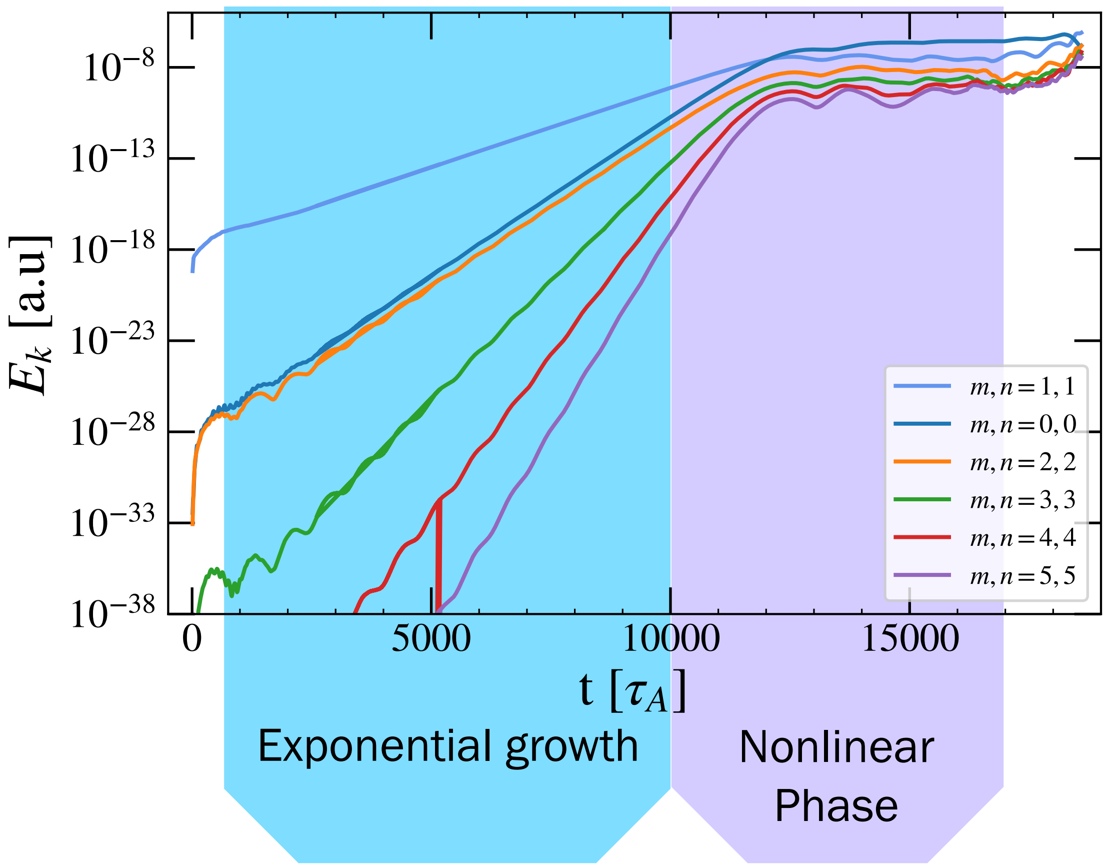


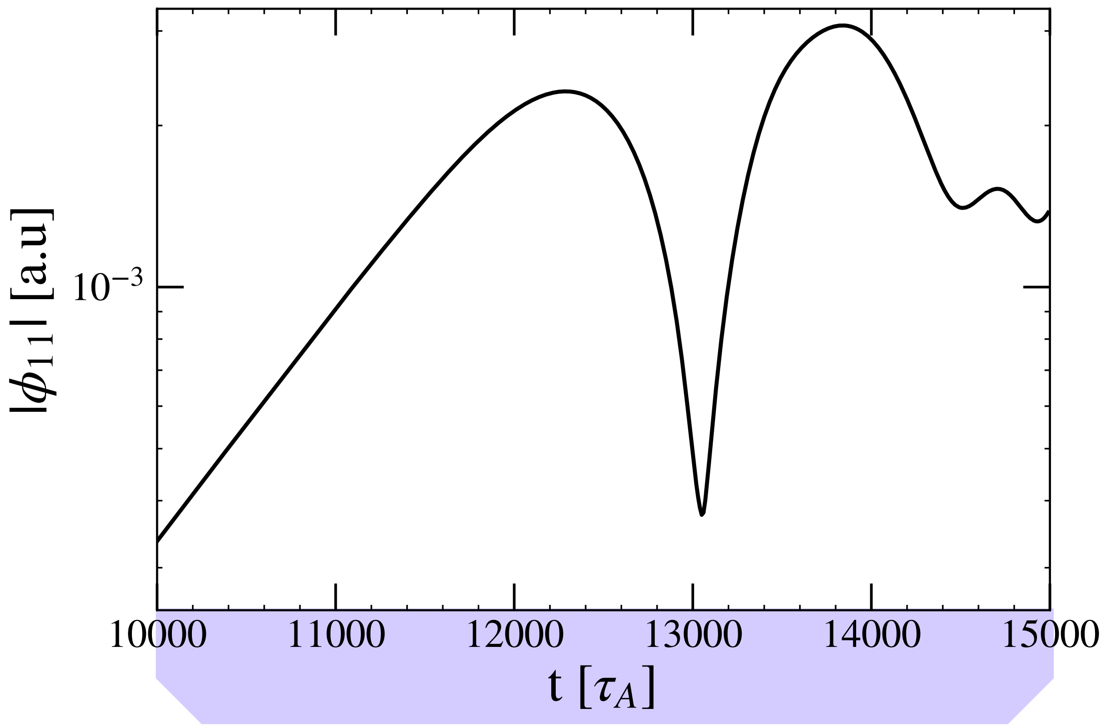


Nonlinear Fishbone (close to the threshold, MHD nonlinear effects : ON)

- Bulk plasma kinetic energy



- Amplitude of the fishbone mode $\phi_{1,1}$

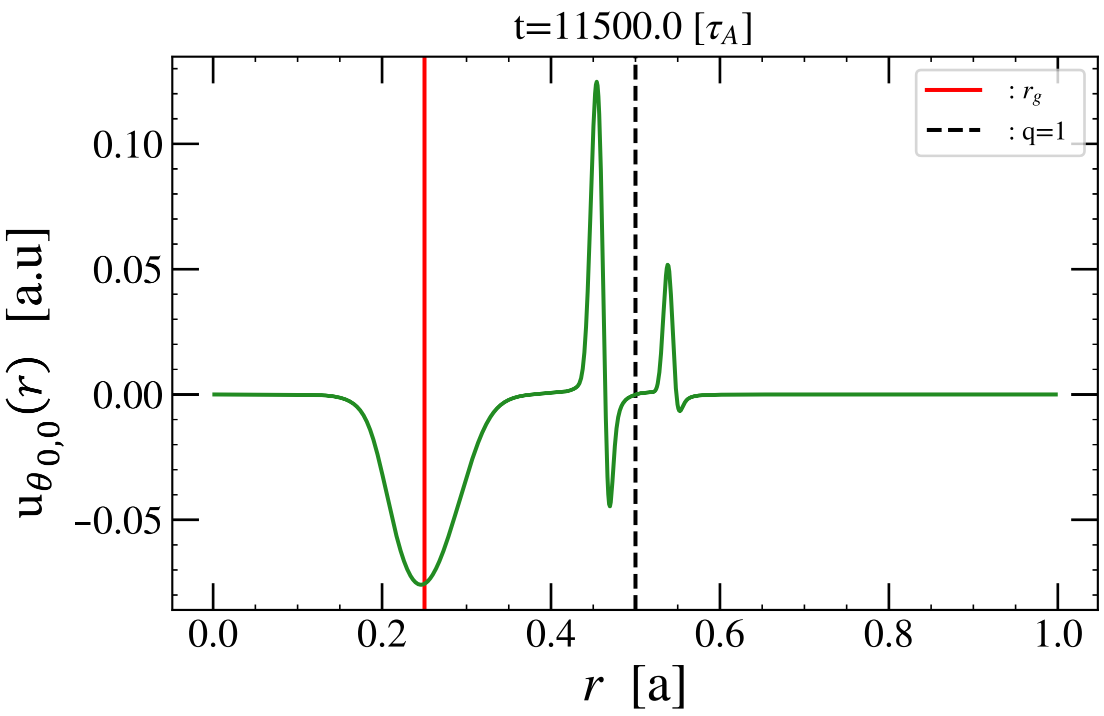


Nonlinear Fishbone (close to the threshold, MHD nonlinear effects : ON)

- Generation of zonal flow

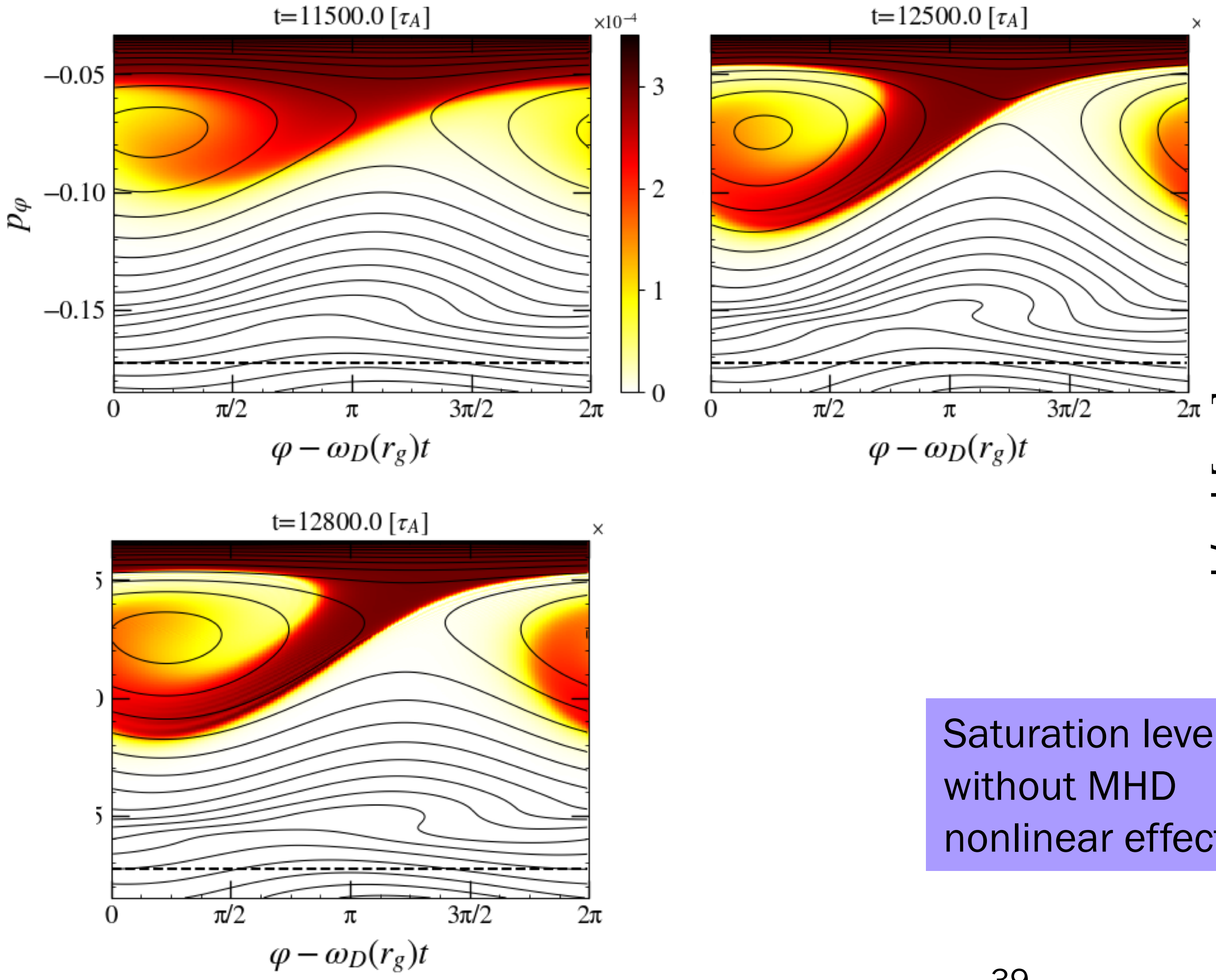
$$u_{\theta\,0,0}(r) = \frac{\partial\,\phi_{0,0}}{\partial r} \quad \Longrightarrow \quad \omega_{ZF}|_{r_g}$$

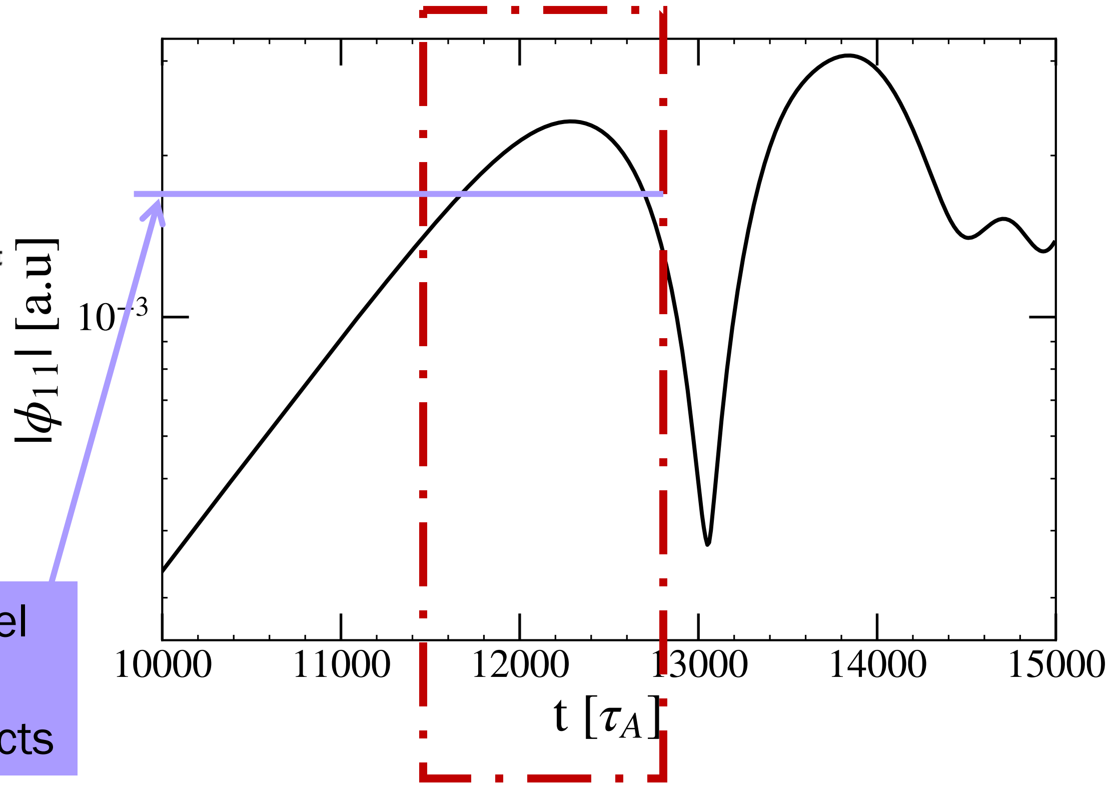
  - Coupling of the mode $m = \pm 1$
  - Developed at linear resonance position r_g
  - Amplitude of $\phi_{0,0}$ dominates the saturation



Energy Exchange between mode and resonant EP

(close to the threshold, MHD nonlinear effects : ON)



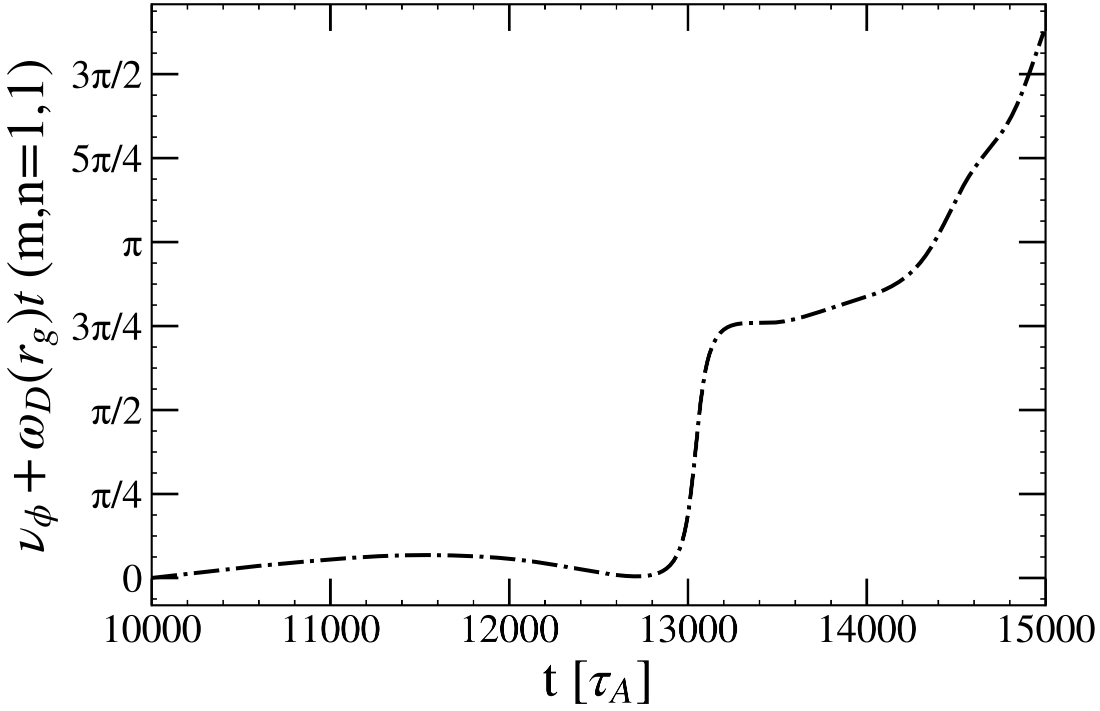


Saturation level without MHD nonlinear effects

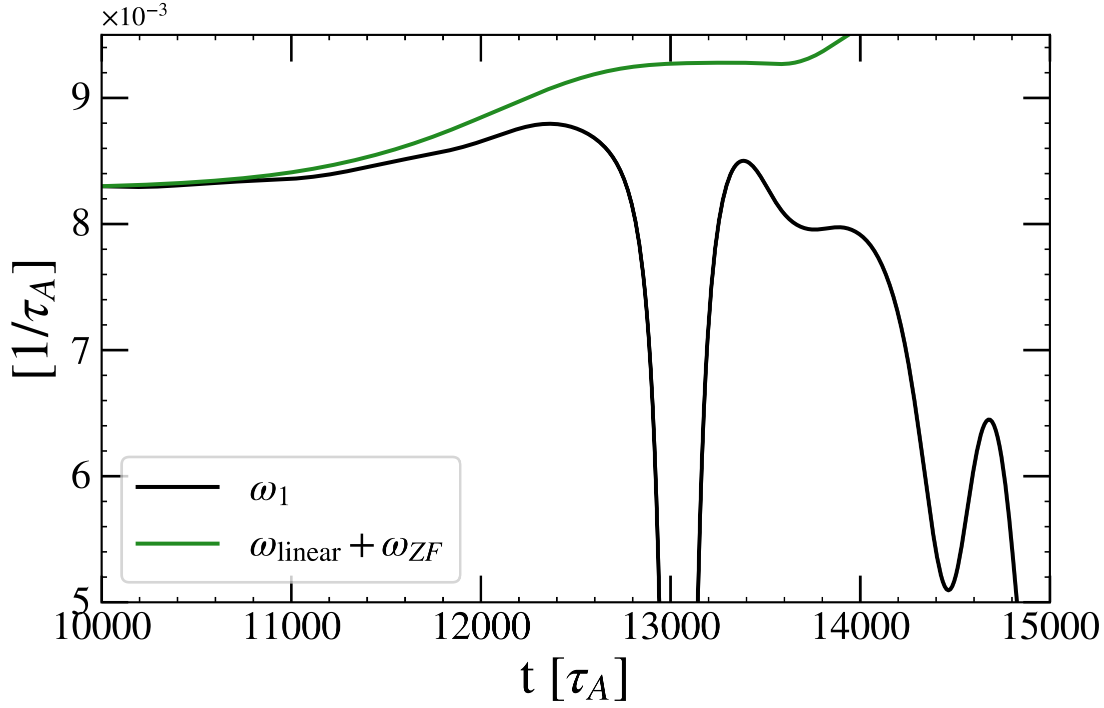
Resonance Condition Maintained

(close to the threshold, MHD nonlinear effects : ON)

- Phase evolution of $\phi_{1,1}$:
  - Frame of the resonant interaction



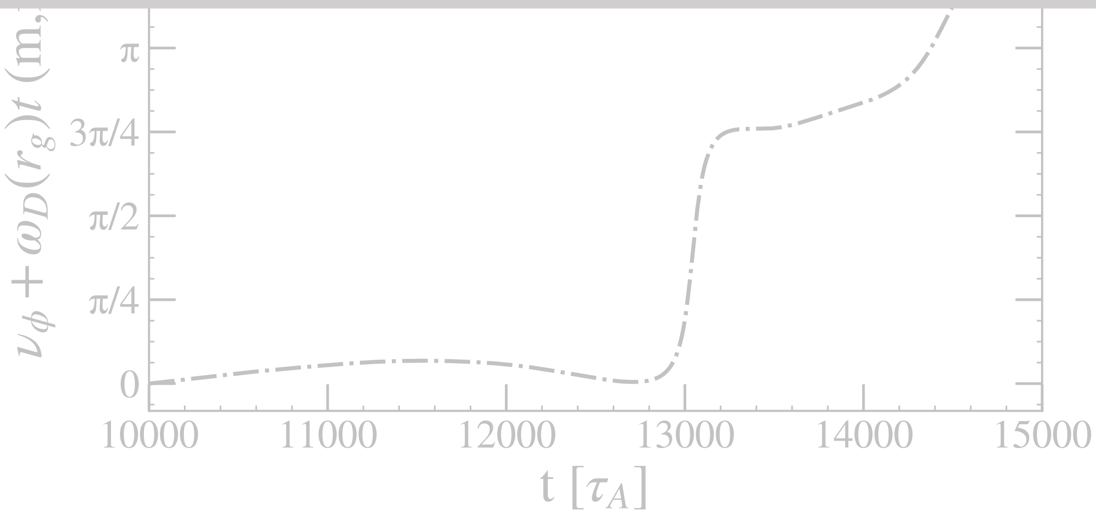
- Evolution of the frequency of $\phi_{1,1}$:



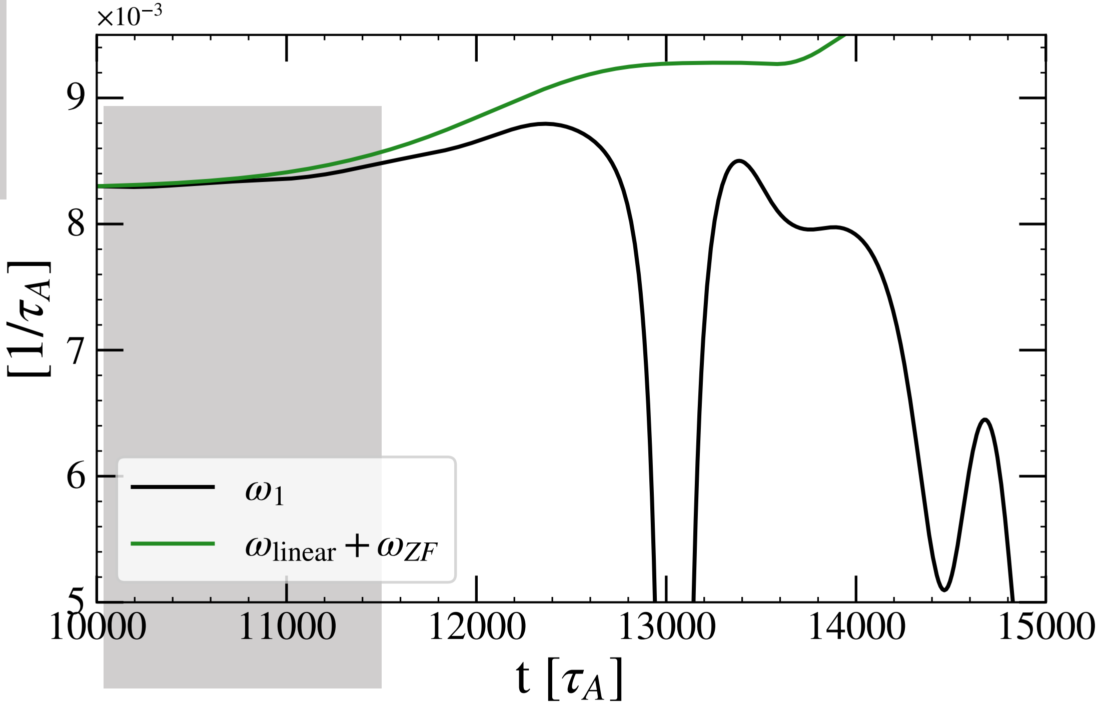
Resonance Condition Maintained

(close to the threshold, MHD nonlinear effects : ON)

- Maitainance of resonance between EP and $\phi_{1,1}$:
  - ▹ No down chirping as $\omega_{1,1}(t)$ seems to follow : $\omega_{\text{linear}}+\omega_{ZF}(t)$
  - ▹ **No** desynchronization



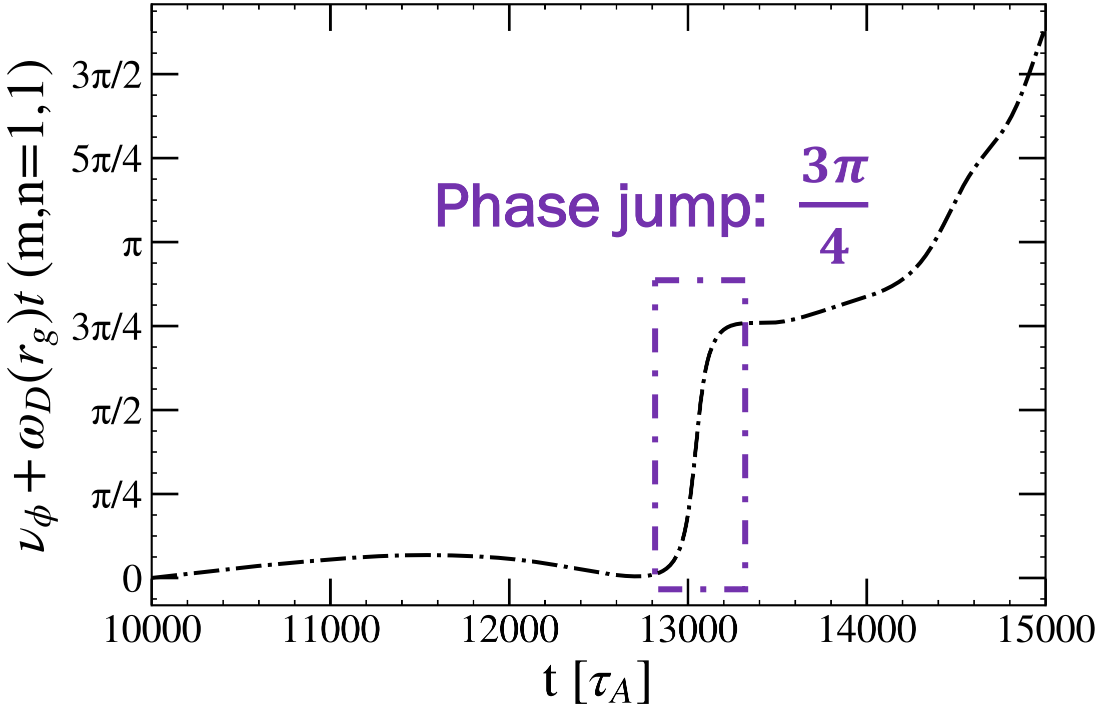
- Evolution of the frequency of $\phi_{1,1}$:

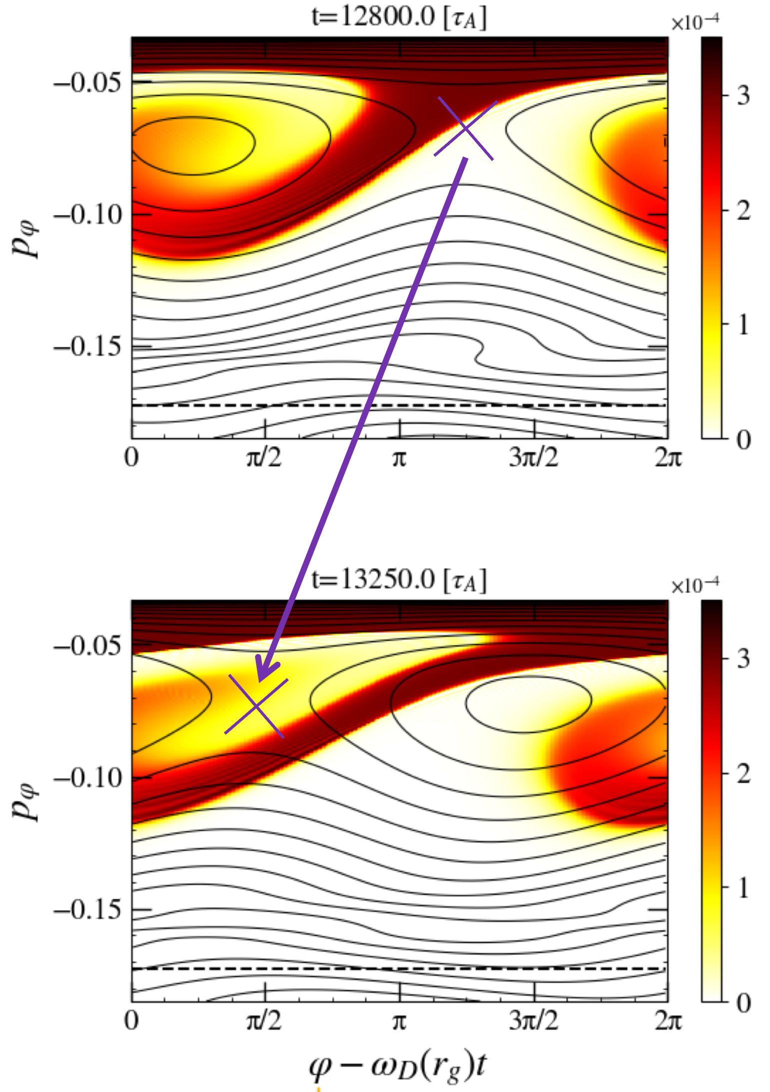


Energy Exchange maximization

(close to the threshold, MHD nonlinear effects : ON)

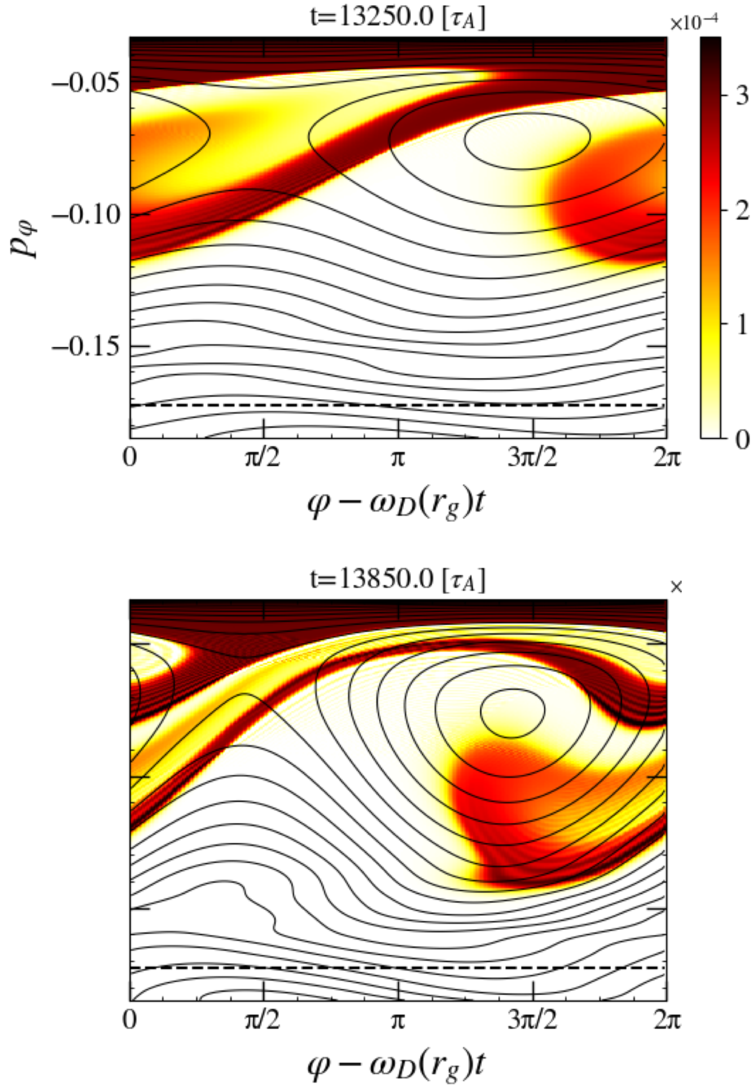
- Phase evolution of $\phi_{1,1}$:
  - Frame of the resonant interaction

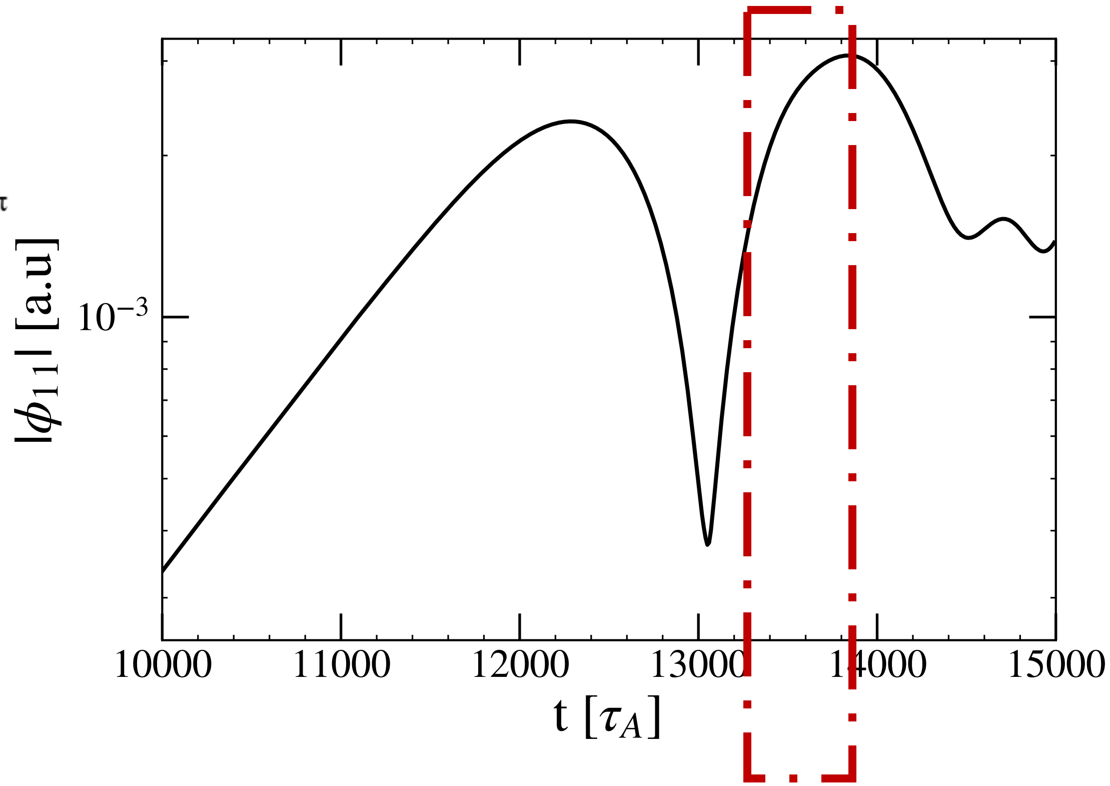




Energy Exchange between mode and resonant EP

(close to the threshold, MHD nonlinear effects : ON)





Zonal Flow Impact on Fishbone Saturation

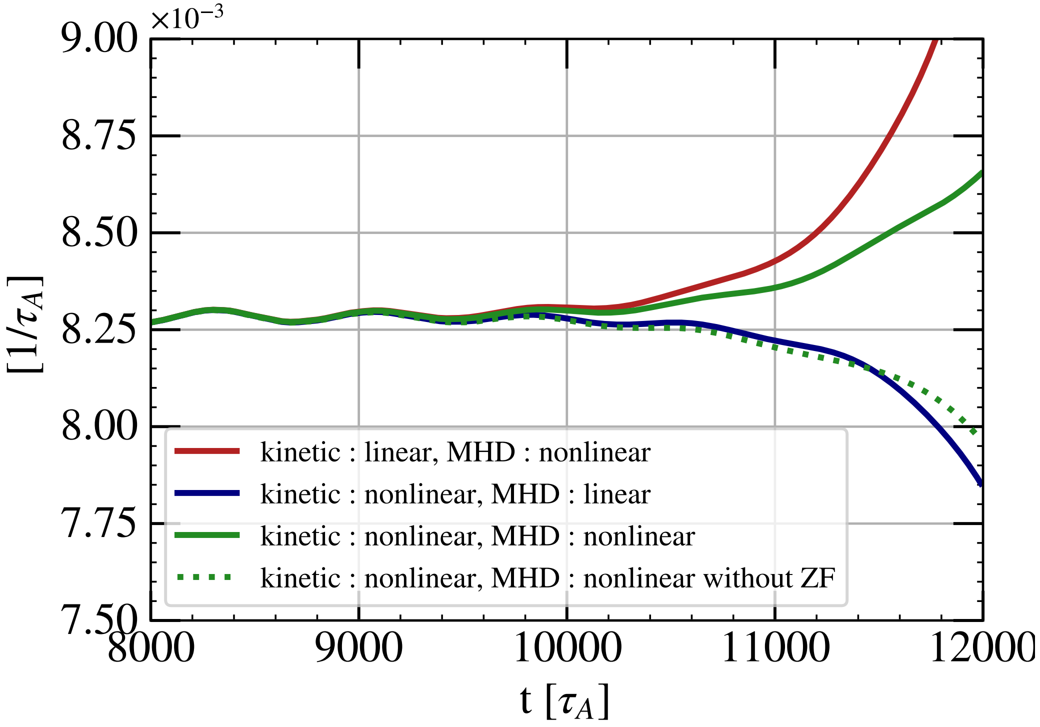
Kinetic : linear, MHD : nonlinear [Odblom, et al., 2002]

Kinetic : nonlinear, MHD : linear

Kinetic : nonlinear, MHD : nonlinear

Kinetic : nonlinear, MHD : nonlinear without ZF

- Evolution of $\omega_{\phi_{1,1}}$ for different types of simulations :



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- ▻ No saturation
- ▻ Frequency explosion

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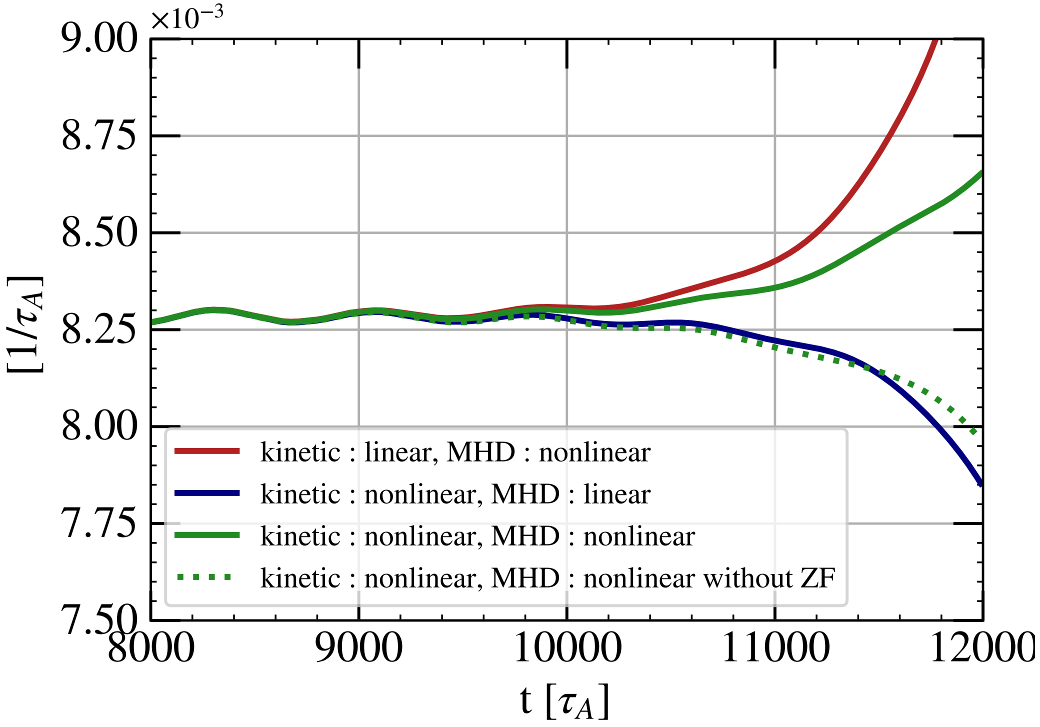

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- ▹ Frequency down-chirping

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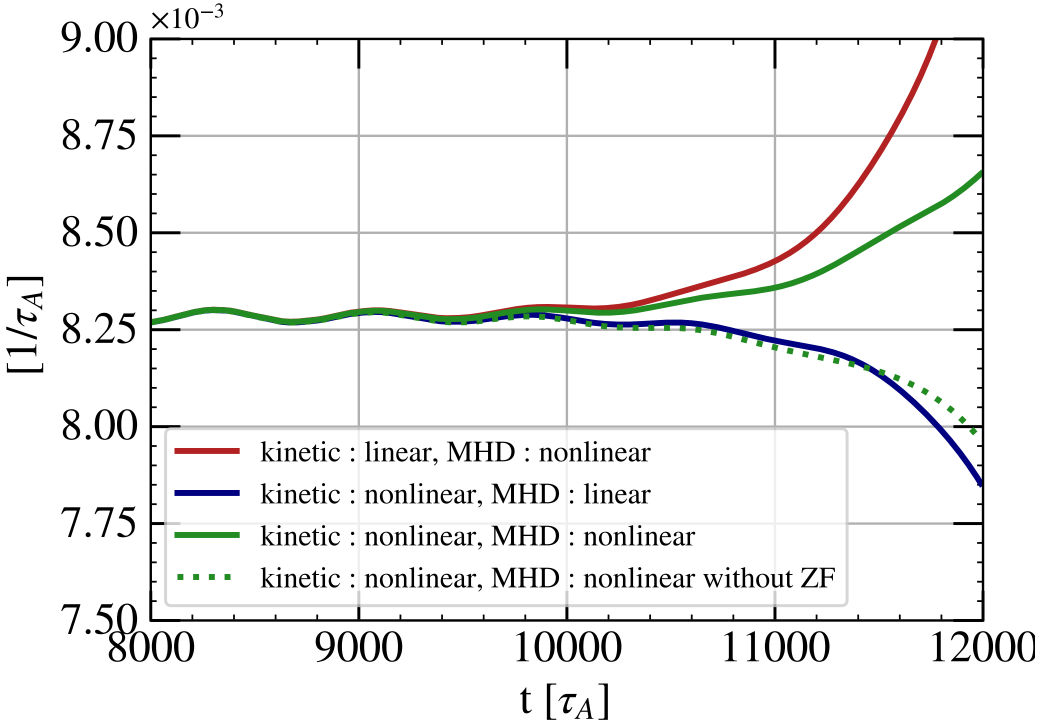
- Evolution of $\omega_{\phi_{1,1}}$ for different types of simulations :



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Kinetic : nonlinear, MHD : nonlinear
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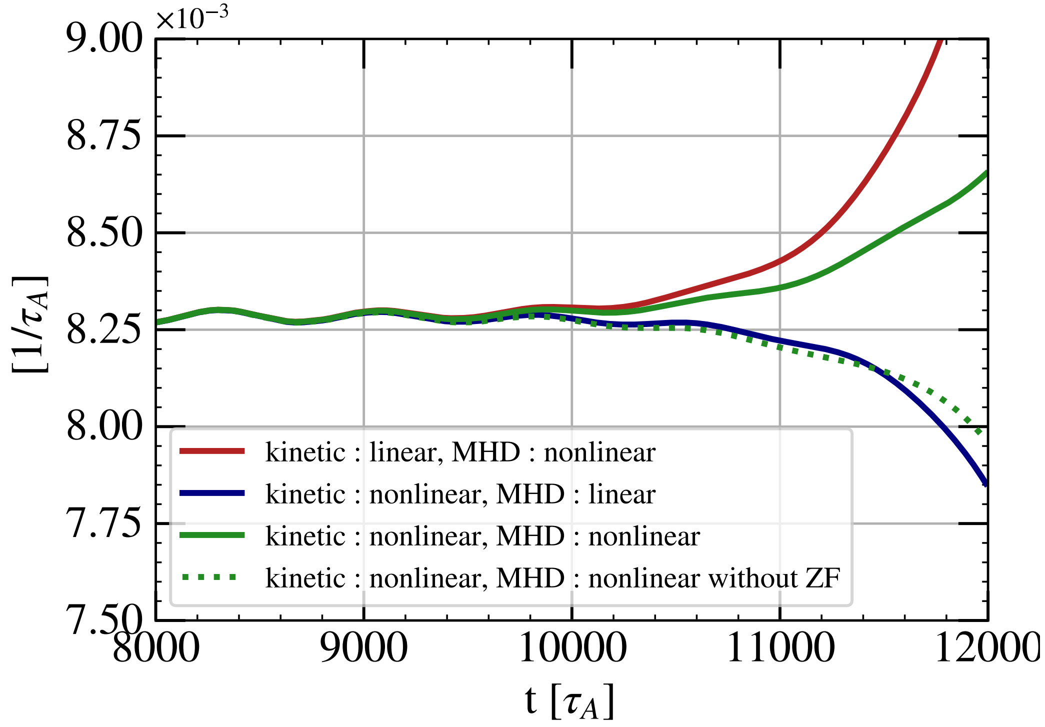
- Evolution of $\omega_{\phi_{1,1}}$ for different types of simulations :



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- ▻ No saturation
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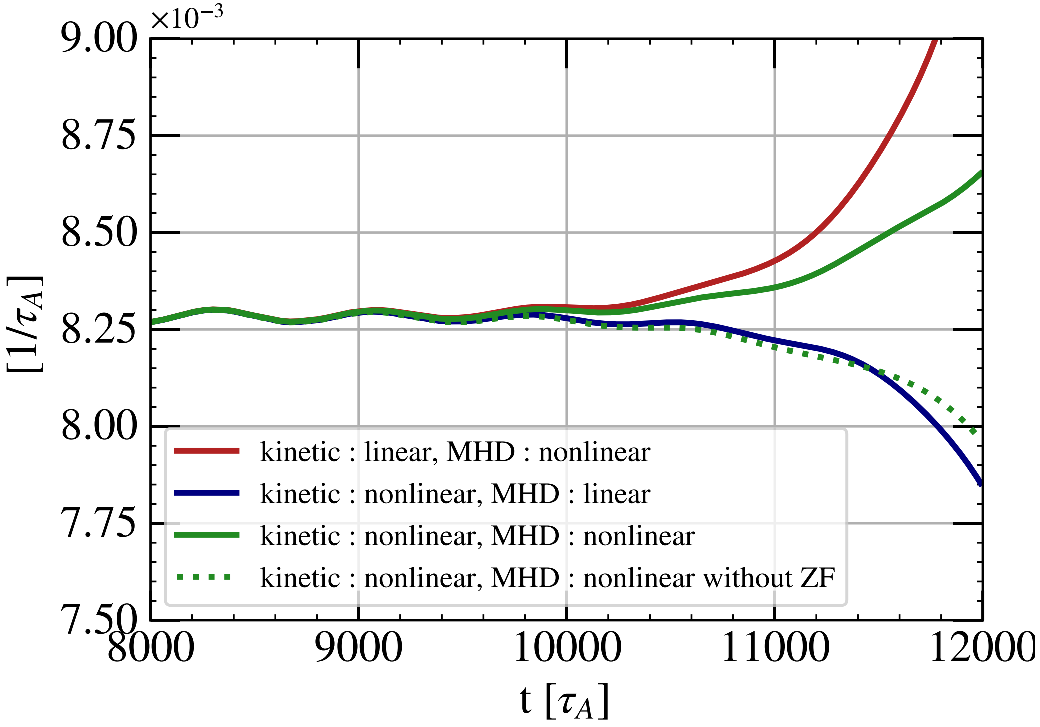
- ▻ Saturation
- ▻ Frequency up-chirping

Kinetic : nonlinear, MHD : nonlinear without ZF

- Evolution of $\omega_{\phi_{1,1}}$ for different types of simulations :



Zonal Flow Impact on Fishbone Saturation

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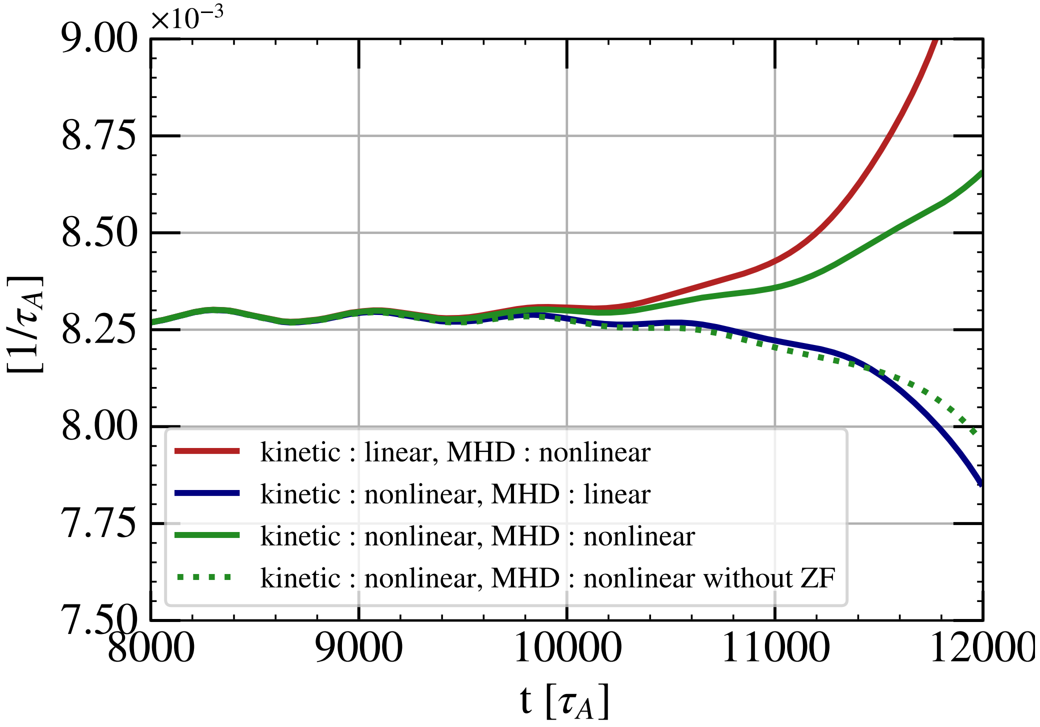
- Evolution of $\omega_{\phi_{1,1}}$ for different types of simulations :



Zonal Flow Impact on Fishbone Saturation

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- ▻ No saturation
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Kinetic : nonlinear, MHD : linear
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- ▻ Saturation
- ▻ Frequency up-chirping

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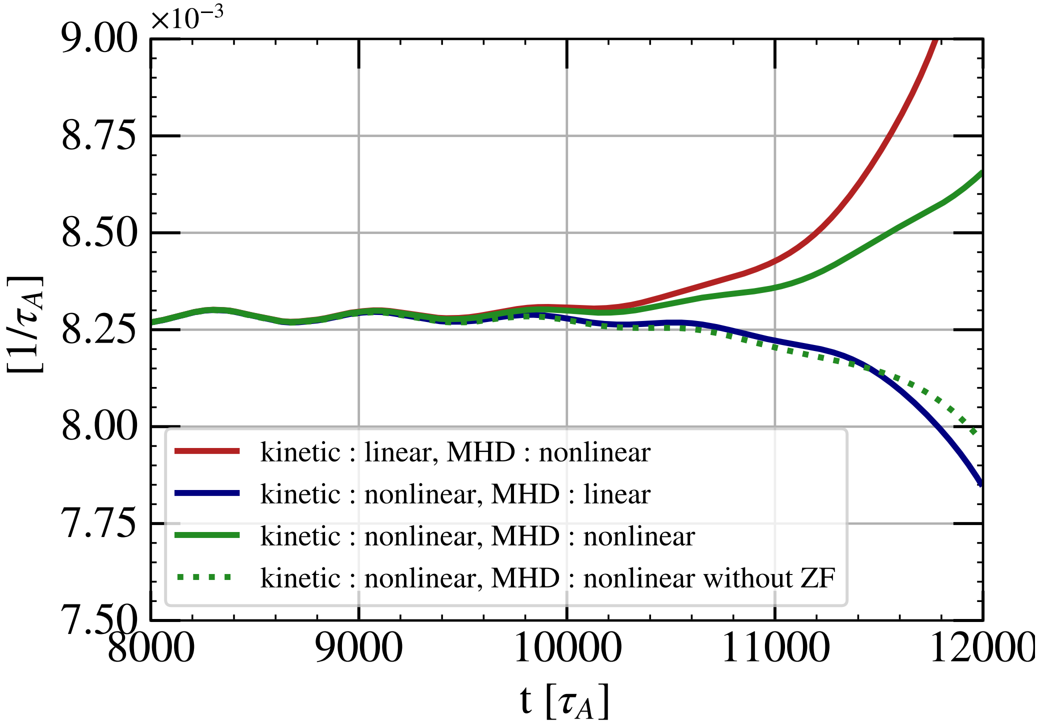
- Evolution of $\omega_{\phi_{1,1}}$ for different types of simulations :



Zonal Flow Impact on Fishbone Saturation

Kinetic : linear, MHD : nonlinear [Odblom, et al., 2002]
- ▻ No saturation
- ▻ Frequency explosion

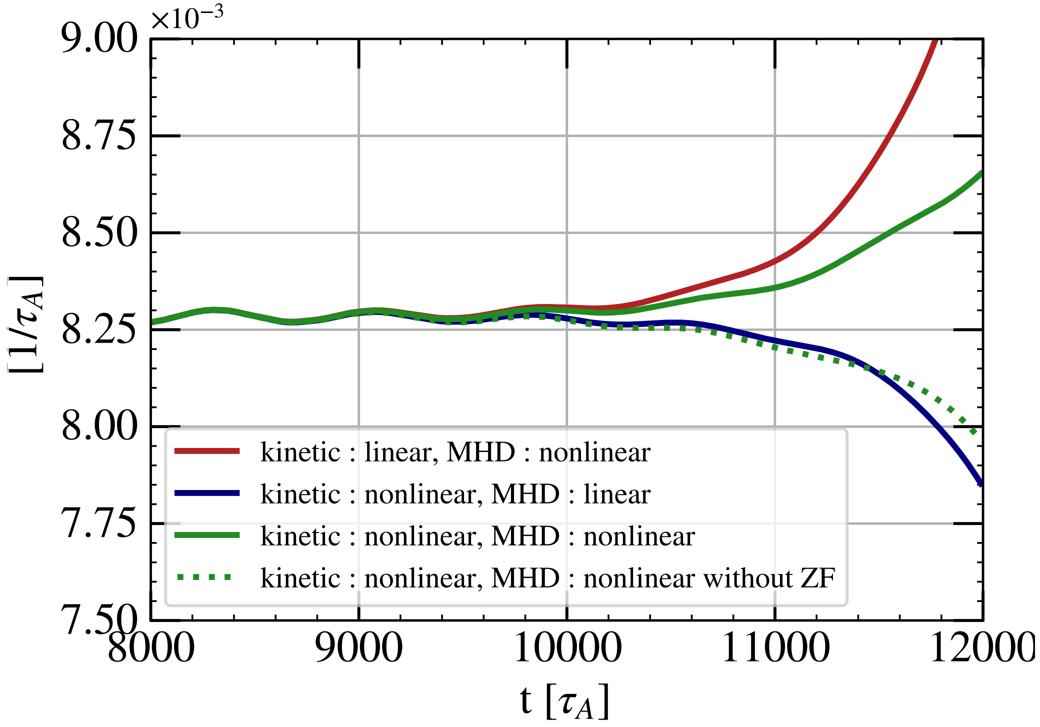
Kinetic : nonlinear, MHD : linear

- MHD nonlinear effect are dominated by the zonal flow growth

Kinetic : nonlinear, MHD : nonlinear
- ▻ Saturation
- ▻ Frequency up-chirping

Kinetic : nonlinear, MHD : nonlinear without ZF
- ▻ Saturation
- ▻ Frequency down-chirping

- Evolution of $\omega_{\phi_{1,1}}$ for different types of simulations :



Conclusion

⇒ Saturation occurs through kinetic nonlinear trapping of EP

⇒ Nonlinear MHD effects are dominated by the growth of the $\phi_{0,0}$ mode

⇒ Nonlinear MHD effects increase the saturation level

⇒ Advection of the fishbone mode by ZF preserves the resonant interaction by preventing a frequency down-chirping

Thank you for your attention

Alodie Grondin-Exbrayat

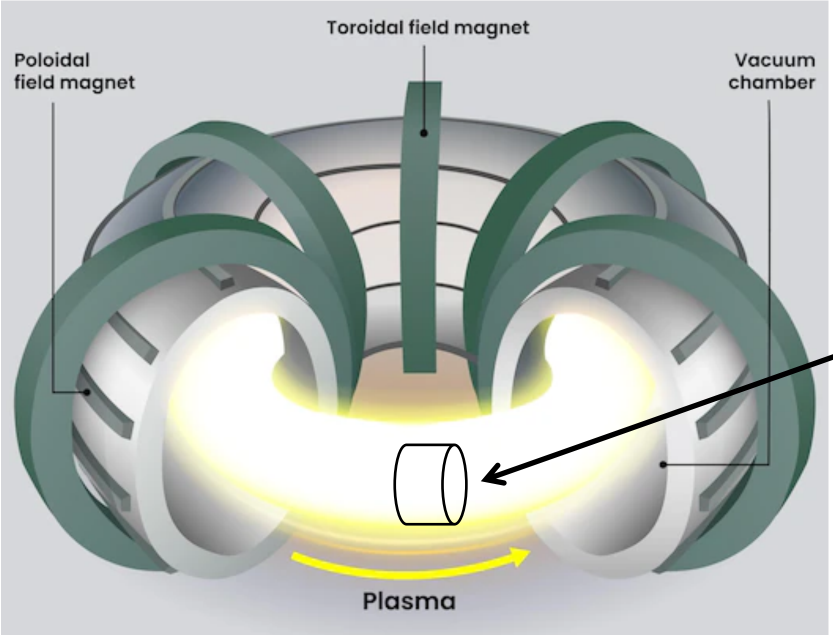
Co authors : Matteo Faganello, Nicolas Dubuit and Alexandre Poyé

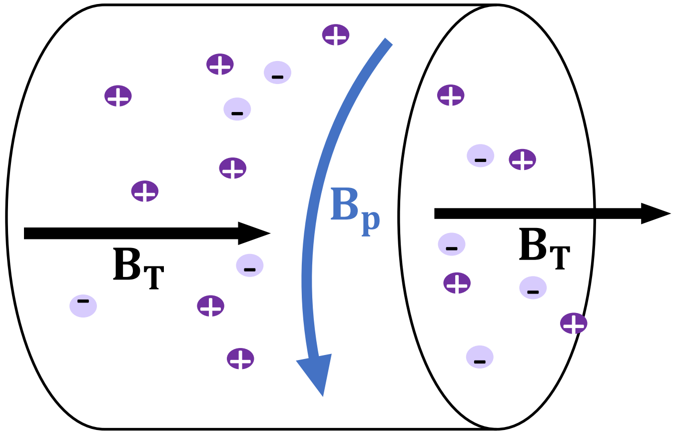
2nd European Conference on Magnetic Reconnection in Plasmas

June 18th, 2025
Turin, Italy

Aix Marseille Université

Tokamak Plasma

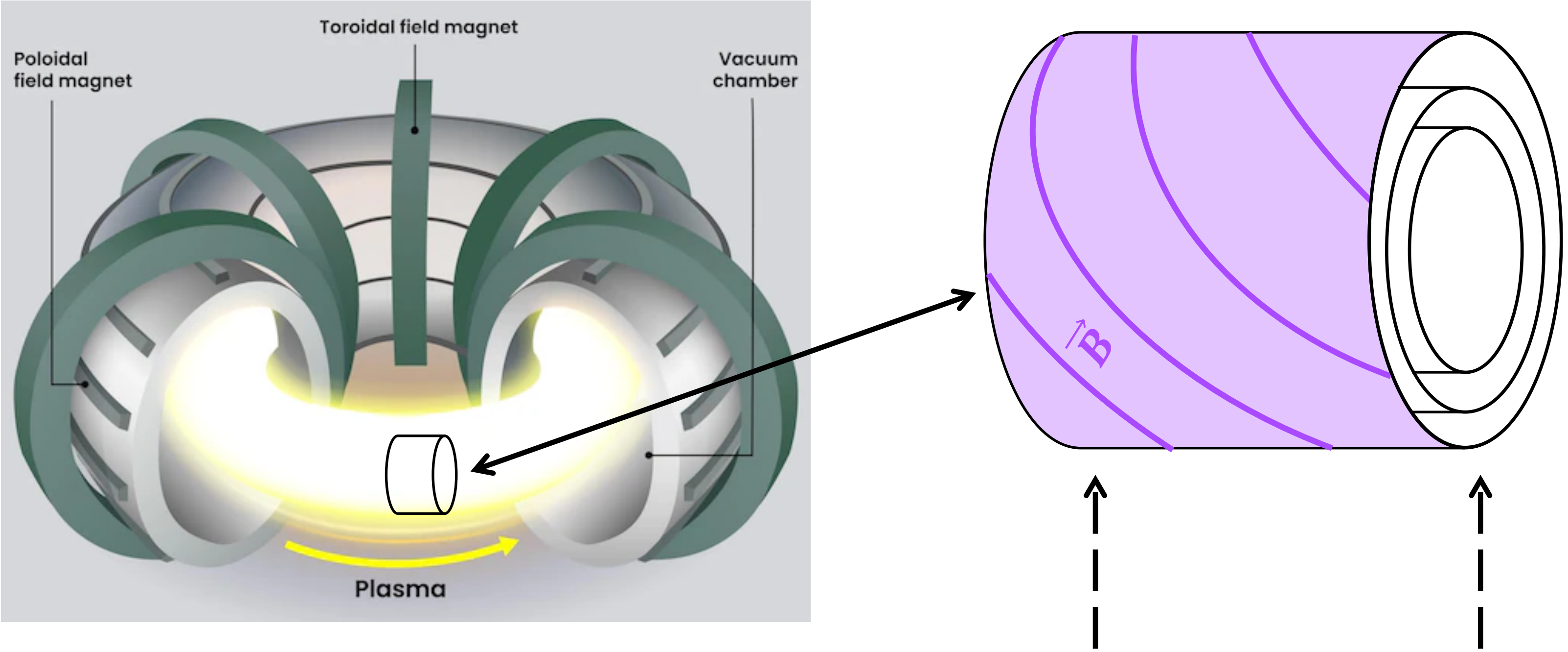




Plasma confined by a « **Toroidal** » and a « **poloidal** » magnetic field .

Typical density in the core : 10^{20} ions/m^3

Tokamak Plasma



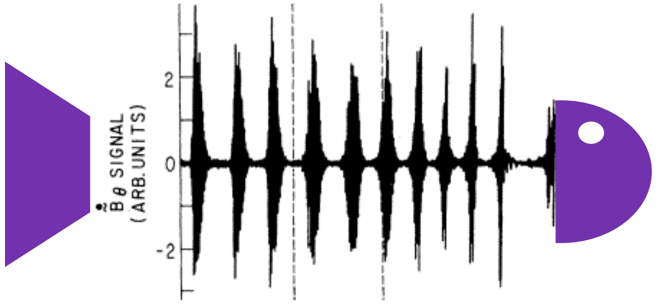
Twisted magnetic field lines

Nested magnetic surfaces = surfaces of constant magnetic flux

Safety factor : $q = \dfrac{\Delta\varphi}{2\pi}$

Overview

Precessional fishbone	Diamagnetic fishbone
-Energetic Particle Mode -Strong drive	-Alfven Eigen mode -Low drive

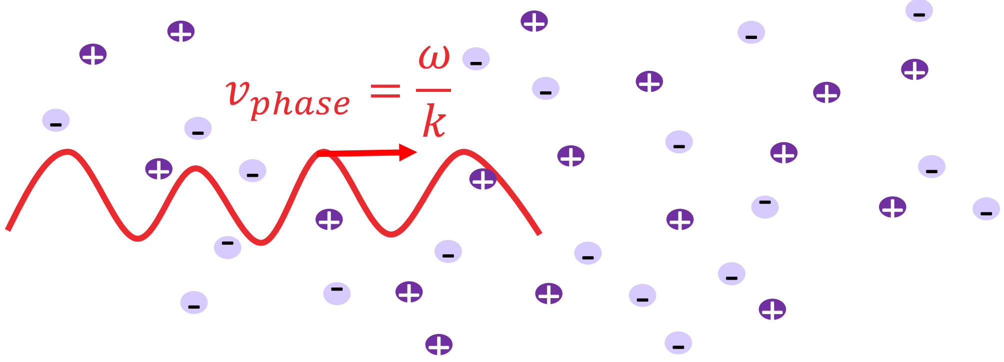


Burst of MHD activity detected by Minrov coils,
McGuire, et al., 1983

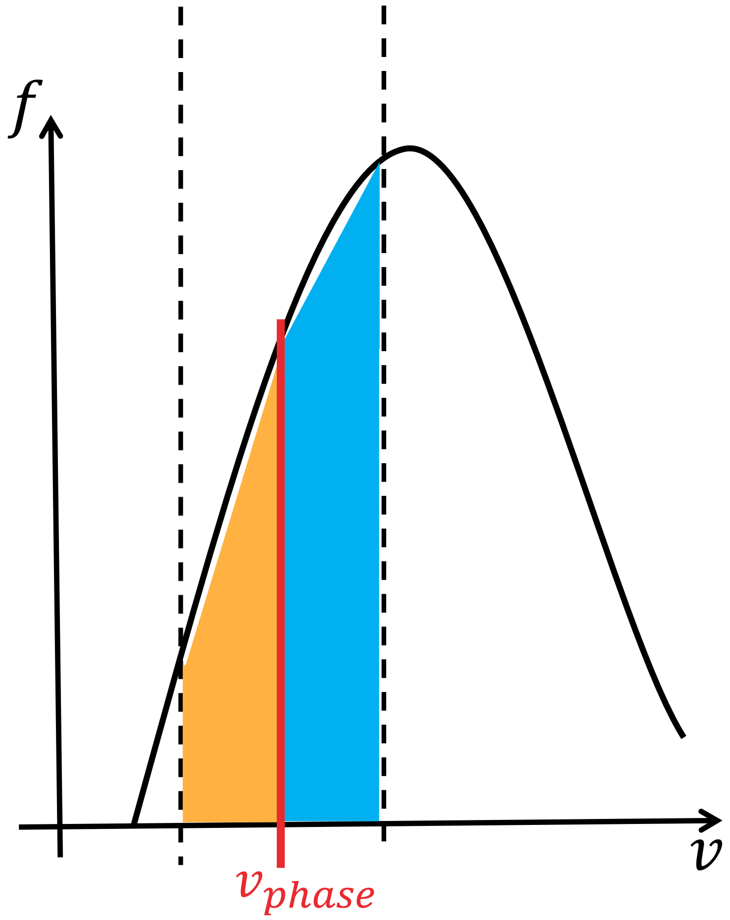
[McGuire, et al., 1983
Coppi, et al., 1986
White, et al. 1989]

Simplified Kinetic Instability

Propagation of a wave

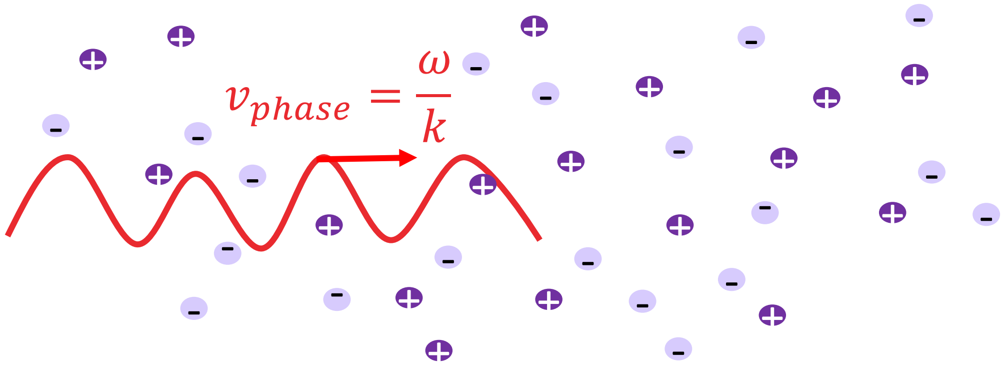


Distribution of velocity



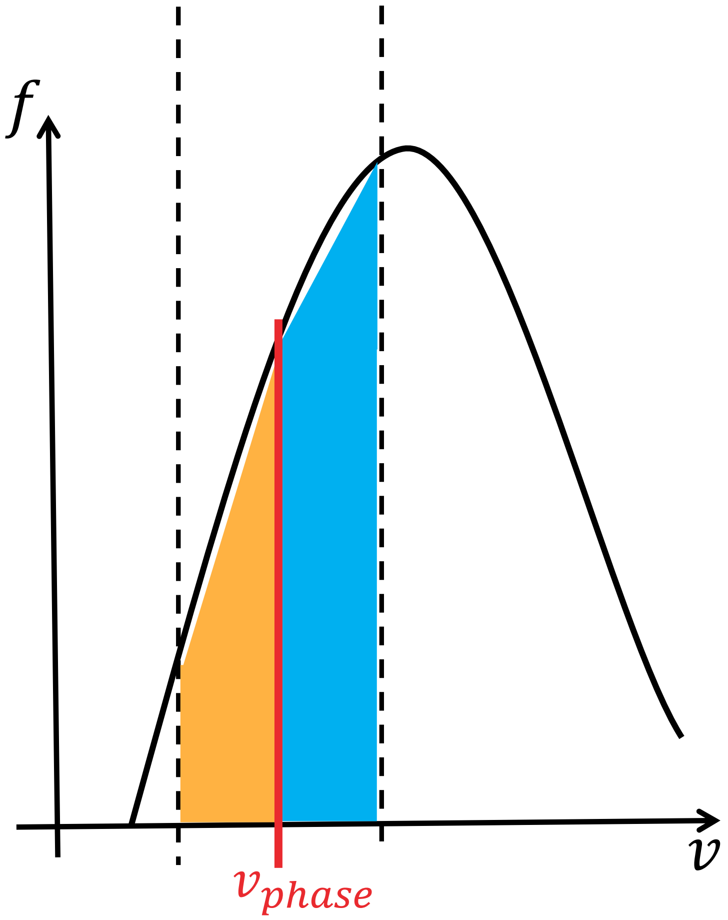
Simplified Kinetic Instability

Propagation of a wave



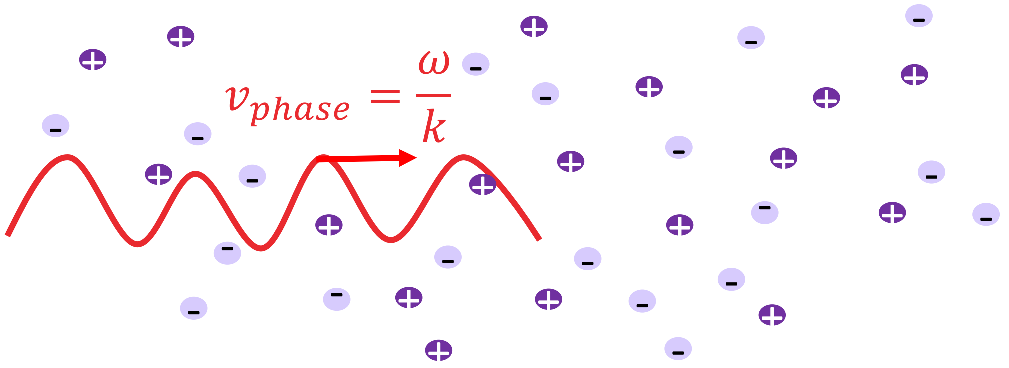
Particles synchronize their velocity with the v_{phase} :

Distribution of velocity



Simplified Kinetic Instability

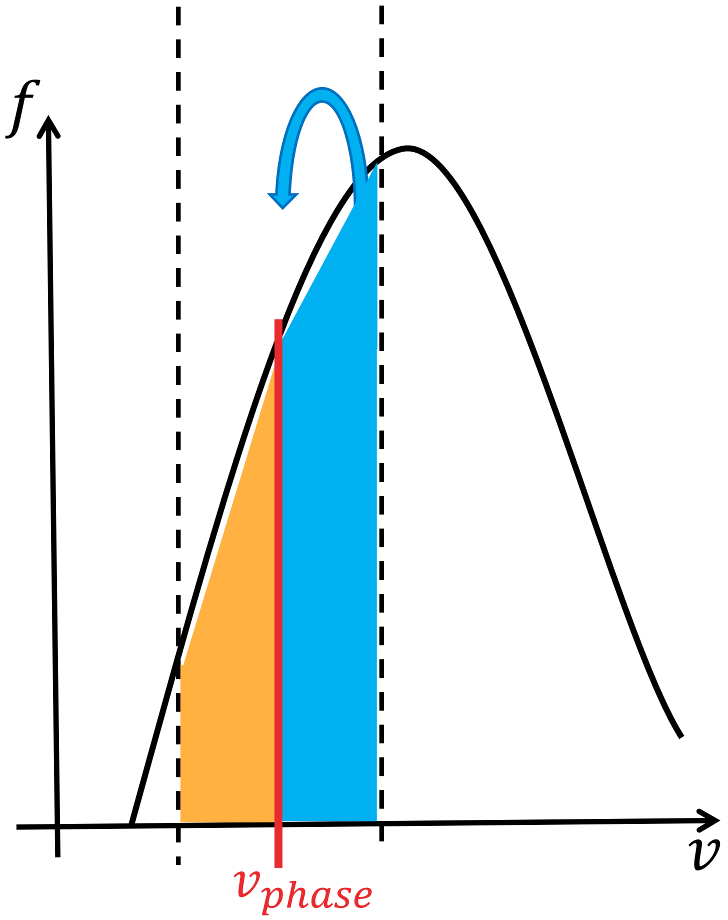
Propagation of a wave



Particles synchronize their velocity with the v_{phase} :

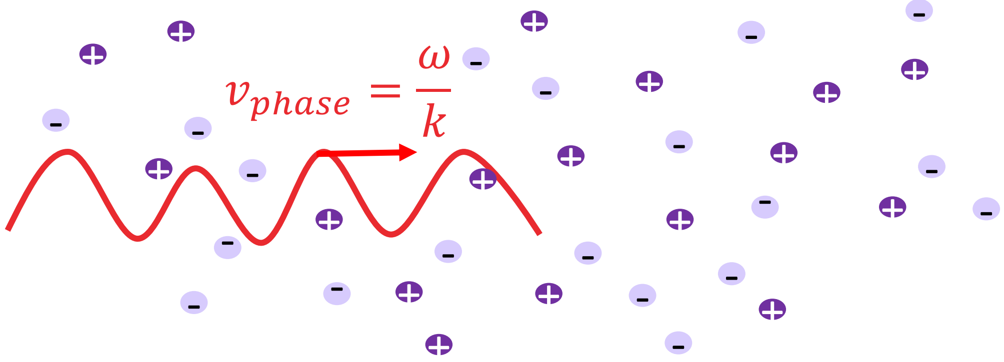
→ Energy gain for the wave (particles decelerate)

Distribution of velocity

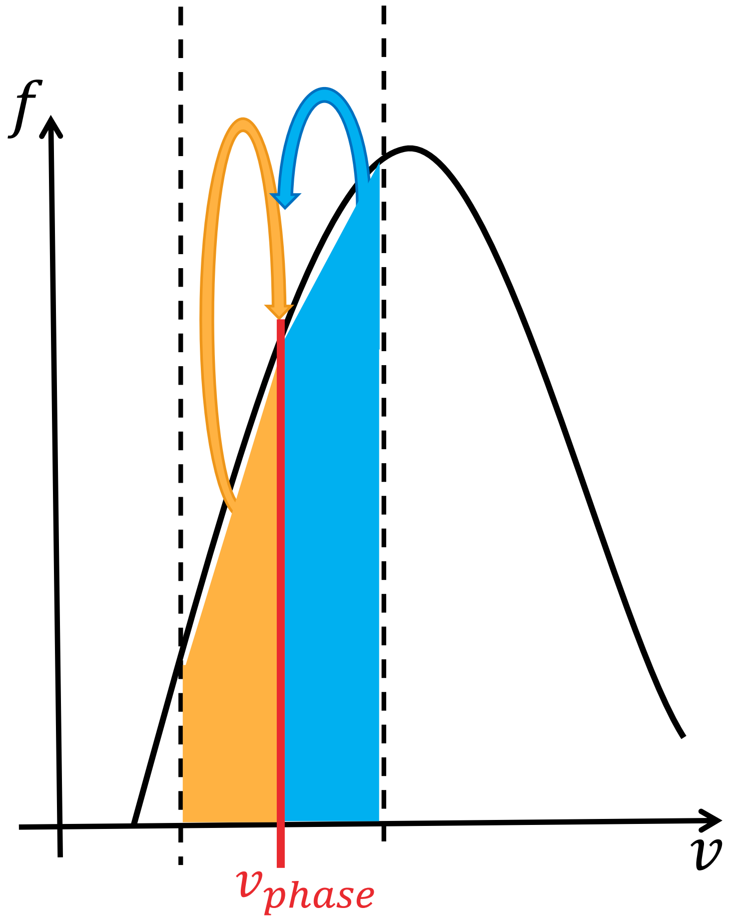


Simplified Kinetic Instability

Propagation of a wave



Distribution of velocity



Particles synchronize their velocity with the v_{phase} :

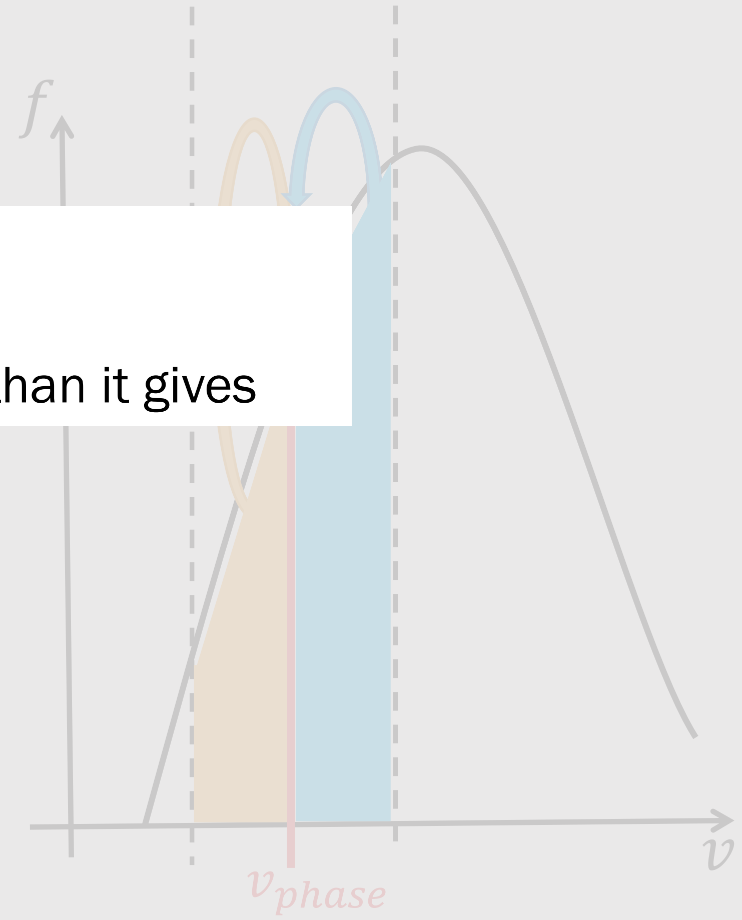
→ Energy gain for the wave (particles decelerate)

→ Energy loss for the wave (particles accelerate)

Simplified Kinetic Instability

Propagation of a wave

Distribution of velocity



Destabilisation of the wave

Since ◢ < ◢ , the wave gains more energy than it gives

Particles synchronize their velocity with the v_{phase} :

→ Energy gain for the wave (particles decelerate)

→ Energy loss for the wave (particles accelerate)

Internal Resistive Kink Mode

without Energetic Particles

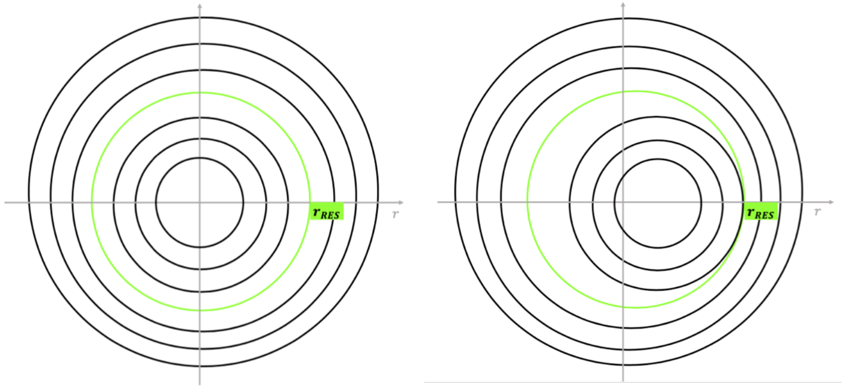
- Constant radial displacement ξ_r of the magnetic surfaces.

- Resonance of the mode $m = n = 1$ at :

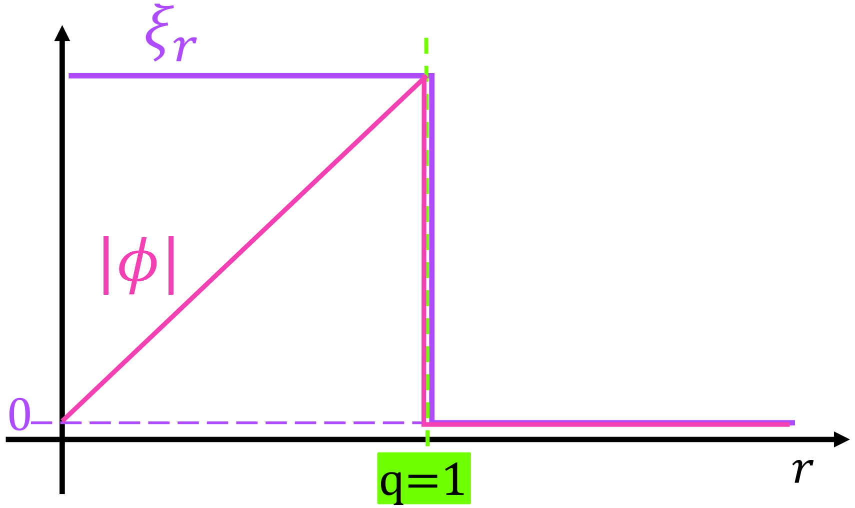
  - $\mathrm{q_{helicity}} \left(= \frac{m}{n}\right) = 1$

- Zero frequency mode and $\gamma \sim \eta^{0.33}$.

[Coppi, et al., 1976,
White review, 1980]



Nested magnetic surfaces



Eigenfunctions

Internal Resistive Kink Mode

without Energetic Particles

Magnetohydrodynamic model ⟶ **Resonance** ⟶ **Kink mode**

$$\begin{cases} \boldsymbol{B} = \mathbf{B}_\varphi + \mathrm{e}_\varphi \times \nabla \Psi \\ \boldsymbol{u} = \dfrac{\mathrm{e}_\varphi \times \nabla \Phi}{B} \end{cases}$$

$$\begin{cases} \phi, \\ \psi, \\ \eta, \\ P = 0 \end{cases}$$

Linear theory :

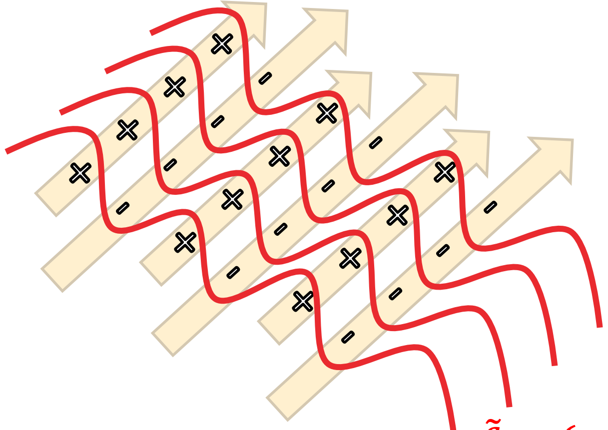
$$A = \mathrm{A_{eq}} + A$$

Perturbation in cylindrical theory :

$$A(r,\theta,\varphi) \Longrightarrow \tilde{A}_{m,n}(r) e^{\,im\theta\,+\,in\varphi} + c.c$$

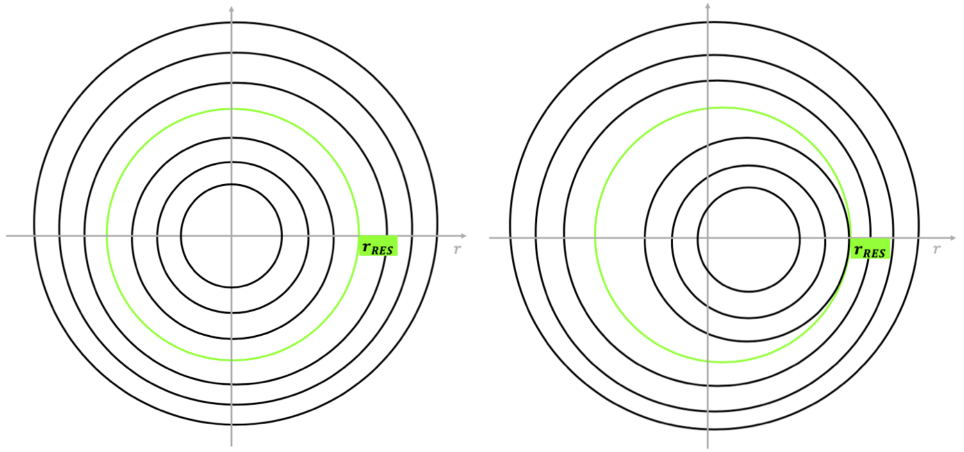
real space *fourier space*

$$q(r_{\mathrm{RES}}) = q_{\mathrm{hecility}\;m,n} = \frac{m}{n} \quad ,$$

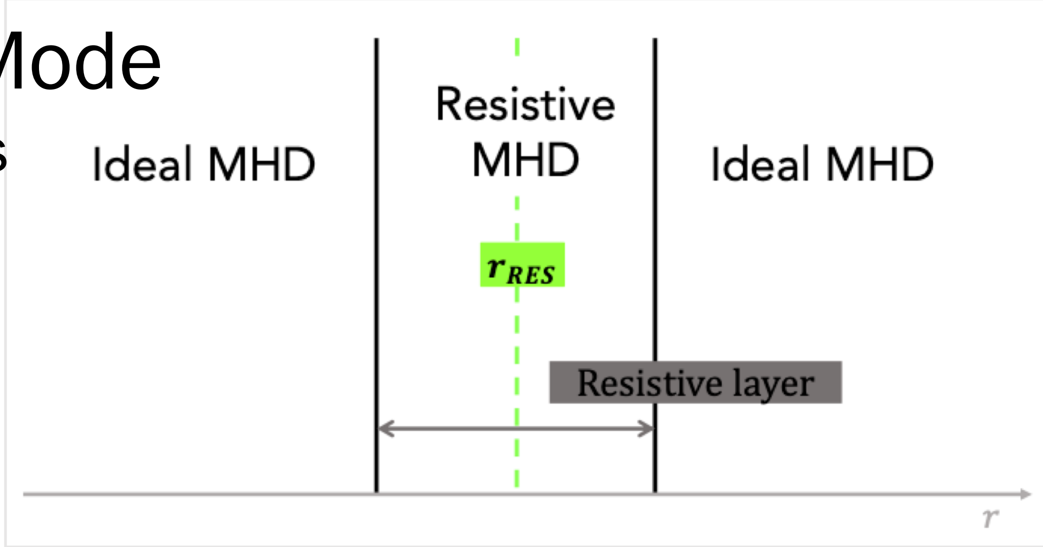


$\tilde{f}_{m,n}(r,\theta,z)$

The kink is defined by a radial displacement ξ_r of the magnetic surface, triggered by the resonance of the mode $\boldsymbol{m = 1, n = 1.}$



Internal Resistive Kink Mode
without Energetic Particles



Using the MHD equations and keeping the term of order one :

$$\frac{\mathrm{d}}{\mathrm{d}r}\left\{\mathrm{r}^3\left[\rho\gamma^2+\left(\frac{m\psi_0'}{r}\right)^2\right]\frac{\mathrm{d}\tilde{\xi}_r}{\mathrm{d}r}\right\}=\left(\left[\frac{(m\psi_0')}{r}+\rho\gamma^2 r\right](m^2-1)+\left(\frac{kr}{m}\right)^2\frac{(\psi_0')^2}{r}\right)\tilde{\xi}_r$$

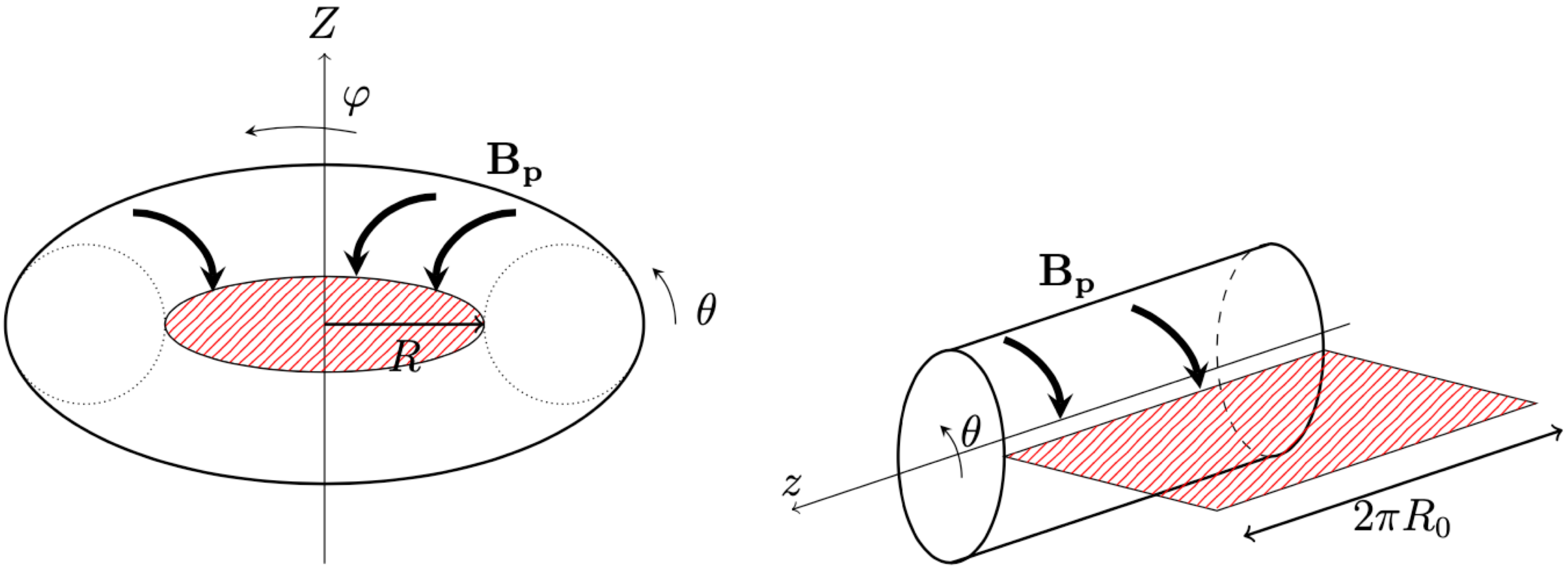
The growth rate of the mode is given by

$$\gamma \sim \int_0^{\mathrm{r_{RES}}} \mathrm{g}\,\mathrm{d}r \longrightarrow \gamma \sim \eta^{0.33}$$

For $r < r_{\mathrm{RES}}$

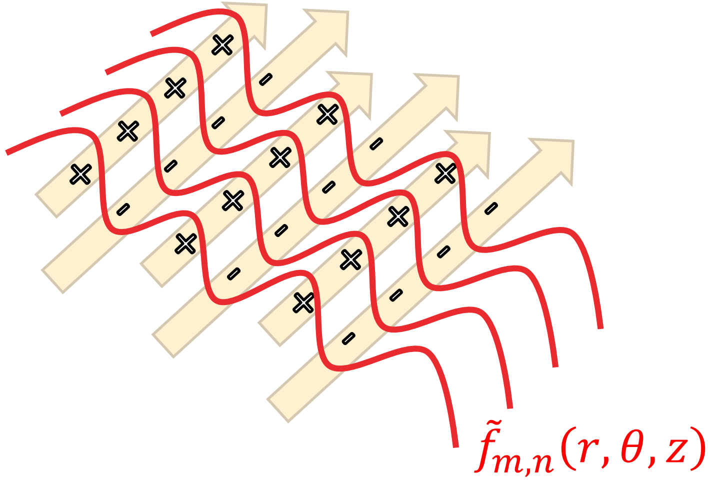
$$\tilde{\xi}_r(r) \sim u_r(r) = \mathrm{cst} \longrightarrow \frac{\phi}{r} = \mathrm{cst}$$

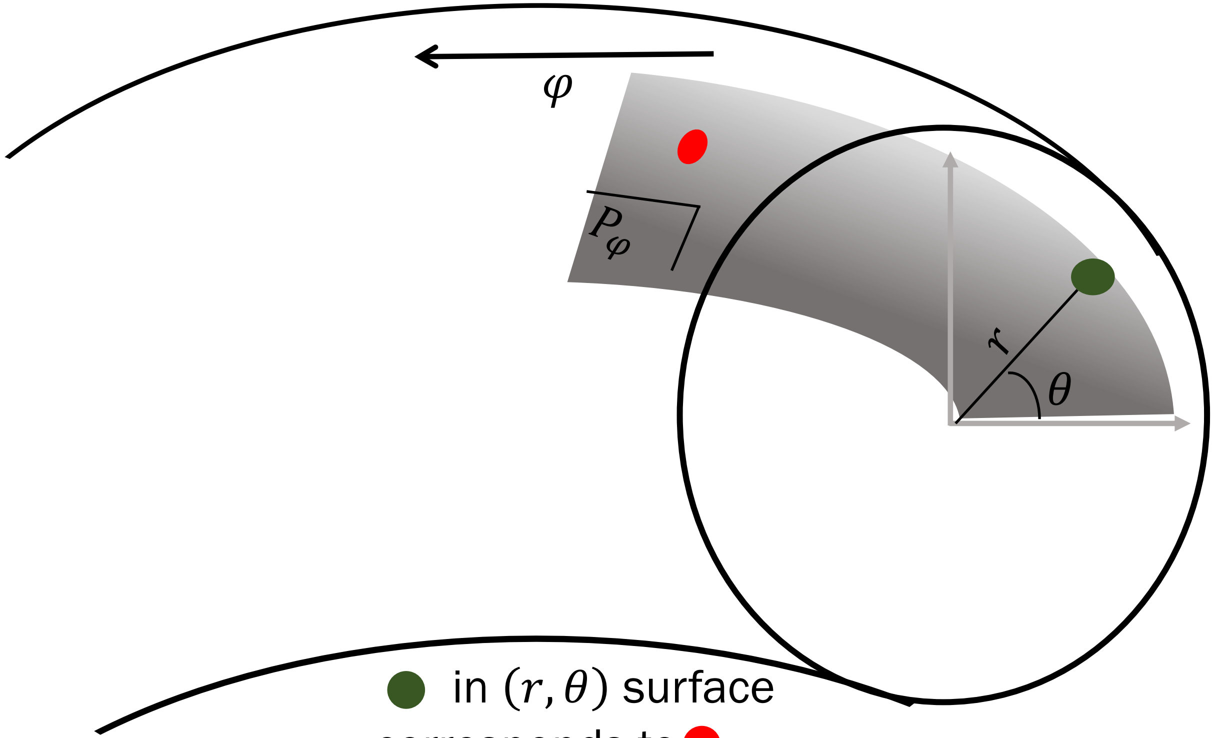
Flux Computation Surfaces



Helicity

The helicity is defined by the direction along which the mode does not vary.

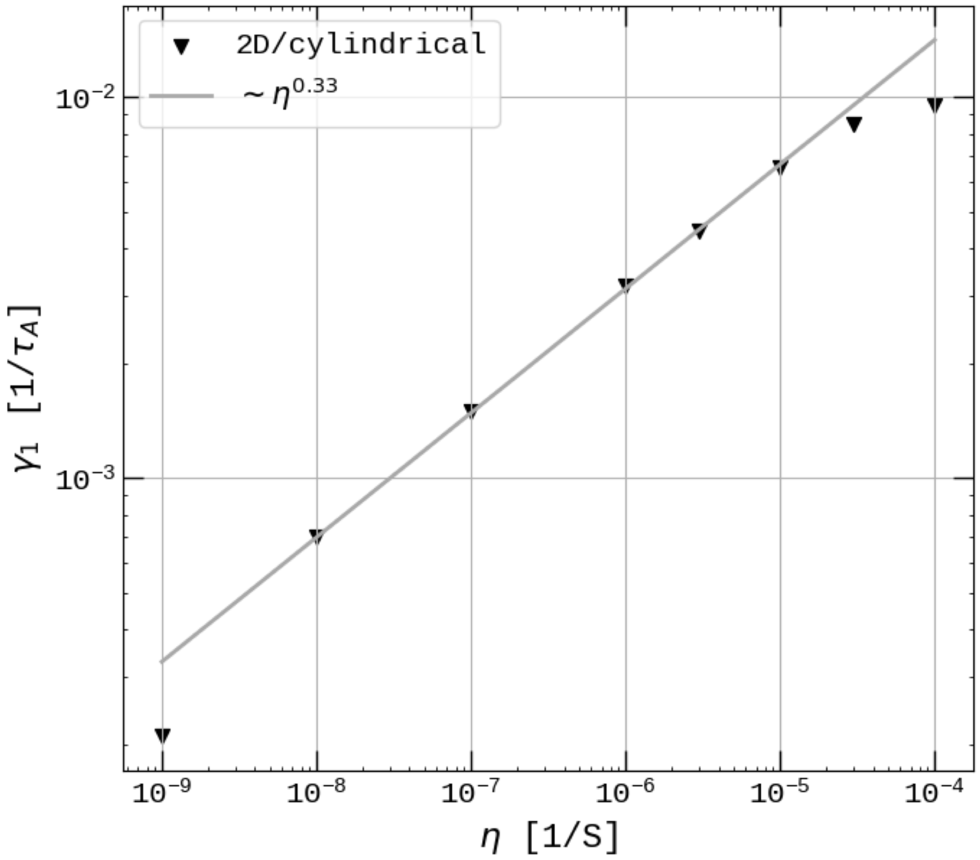




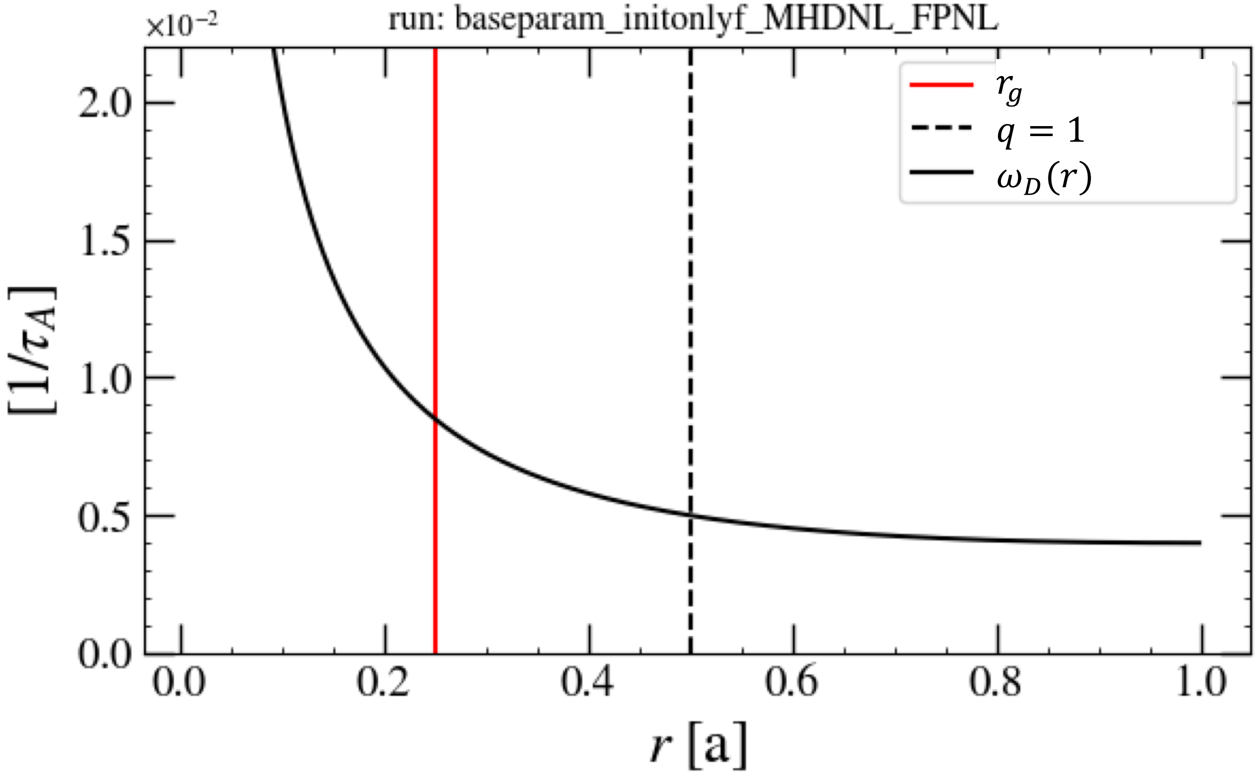
in (r,θ) surface corresponds to in (φ,P_φ) surface

Simulation Setup

- Benchmark of the resistive internal kink without EP : $\gamma \sim \eta^{0.33}$ [Coppi et al. 1976]



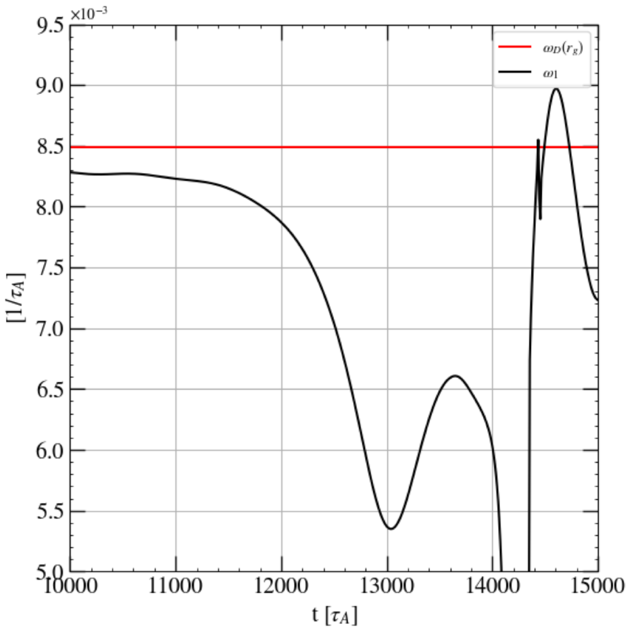
EP precessional frequency profile



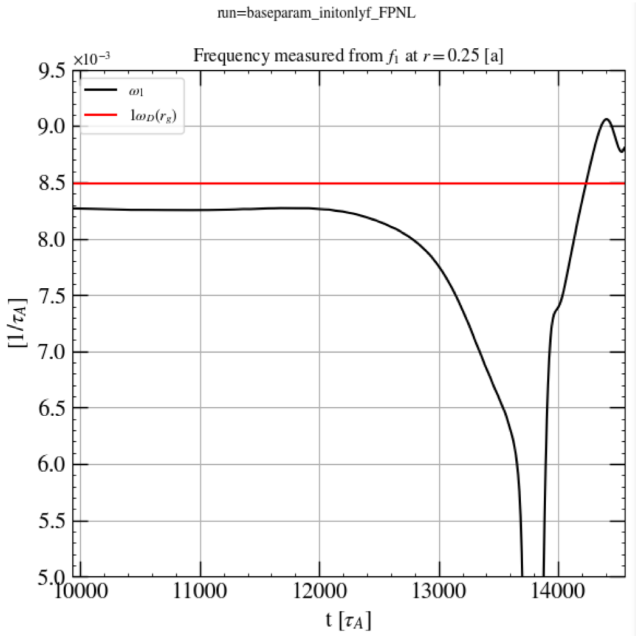
Mode and resonant EP frequency evolution : Down chirping

$\frac{\gamma}{\omega} \simeq 10^{-1}$

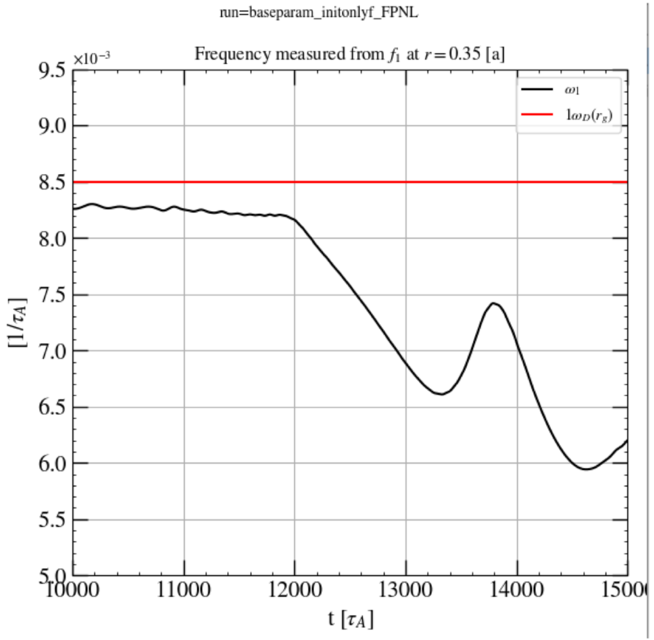
Measured on $\phi_{1,1}$



Measured on $f_{1,1}$



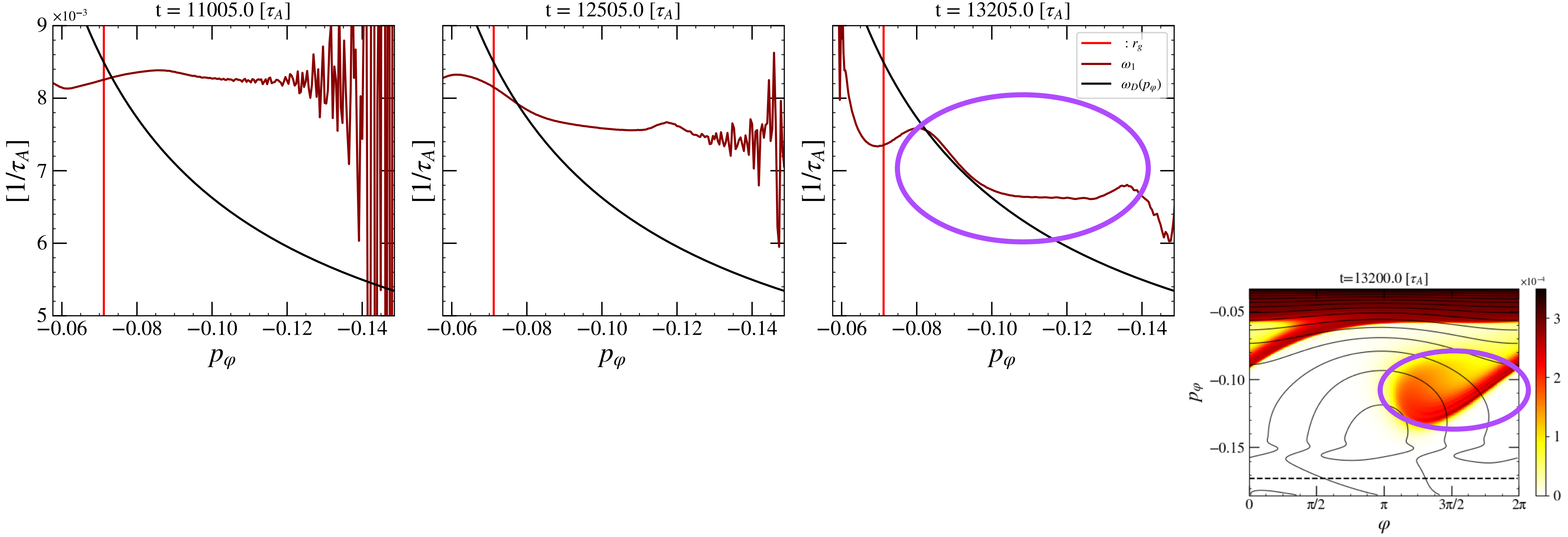
Measured on $f_{1,1}$



Mode and resonant EP frequency evolution : Desynchronization

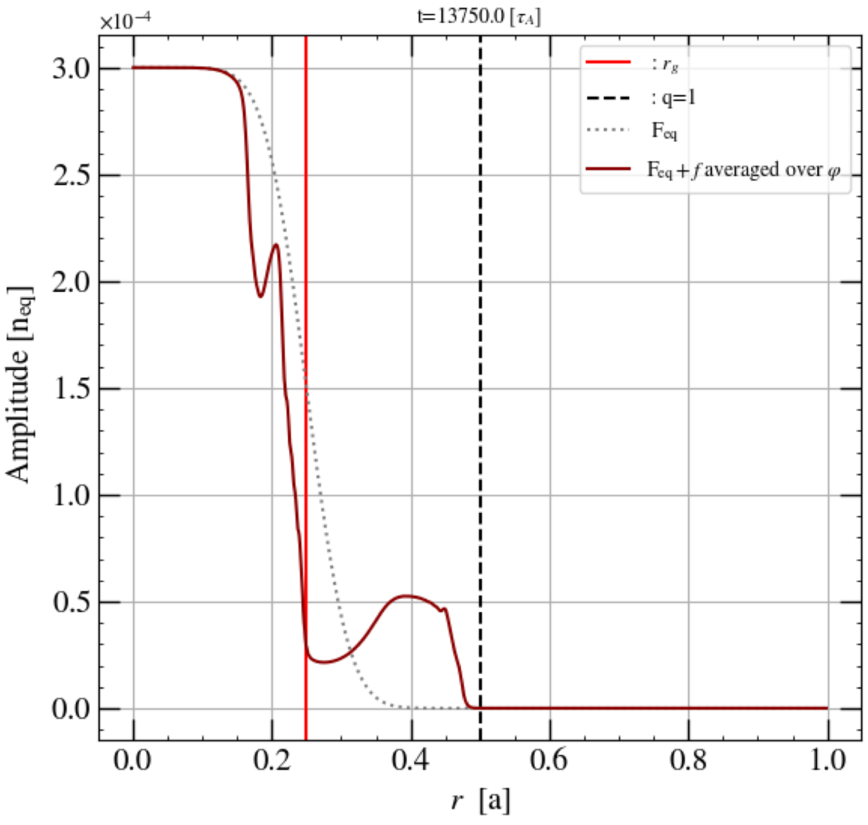
$$\frac{\gamma}{\omega} \simeq 10^{-1}$$

Measured on $f_{1,1}$

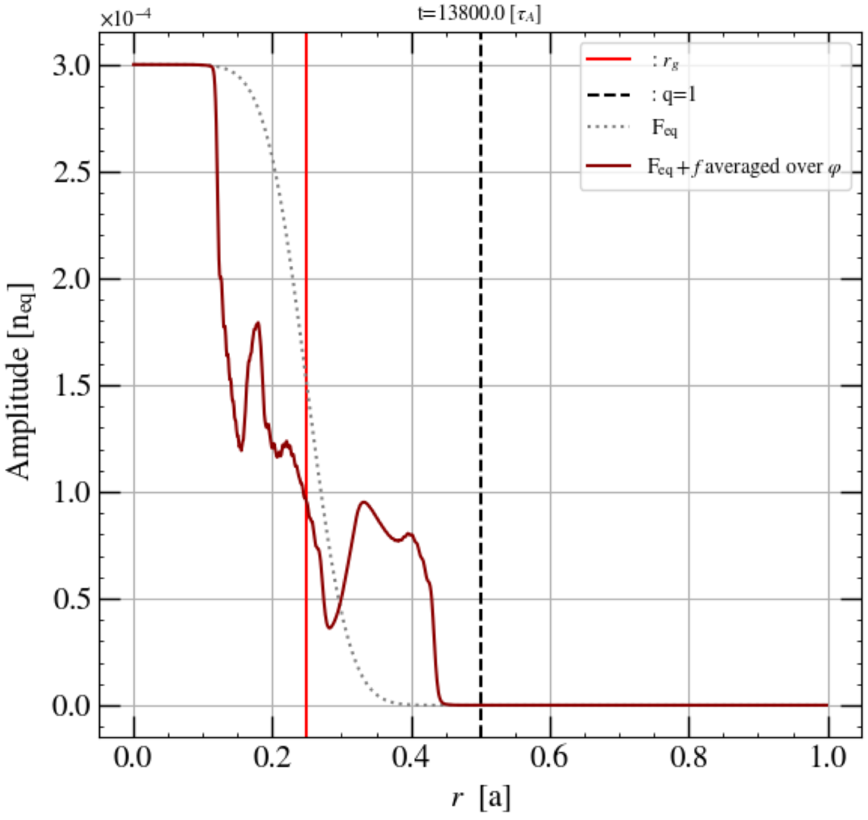


EP transport for $\frac{\gamma}{\omega} \simeq 10^{-1}$

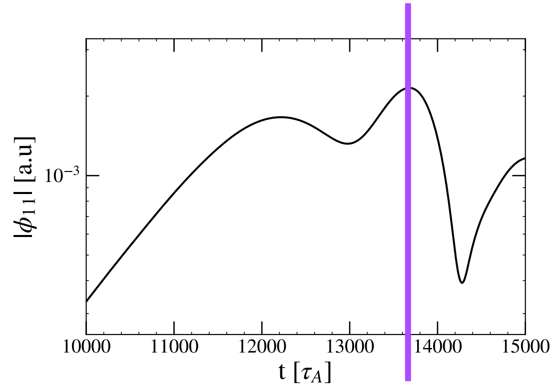
Kinetic : nonlinear, MHD : linear



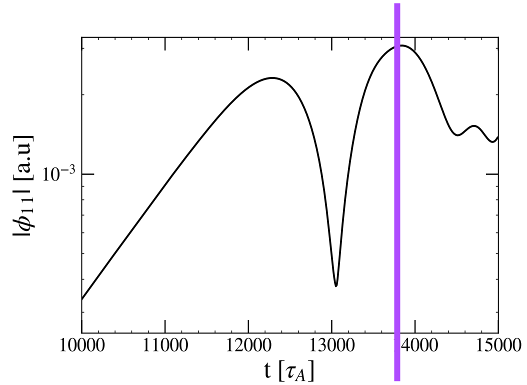
Kinetic : nonlinear, MHD : nonlinear



Kinetic : nonlinear, MHD : linear

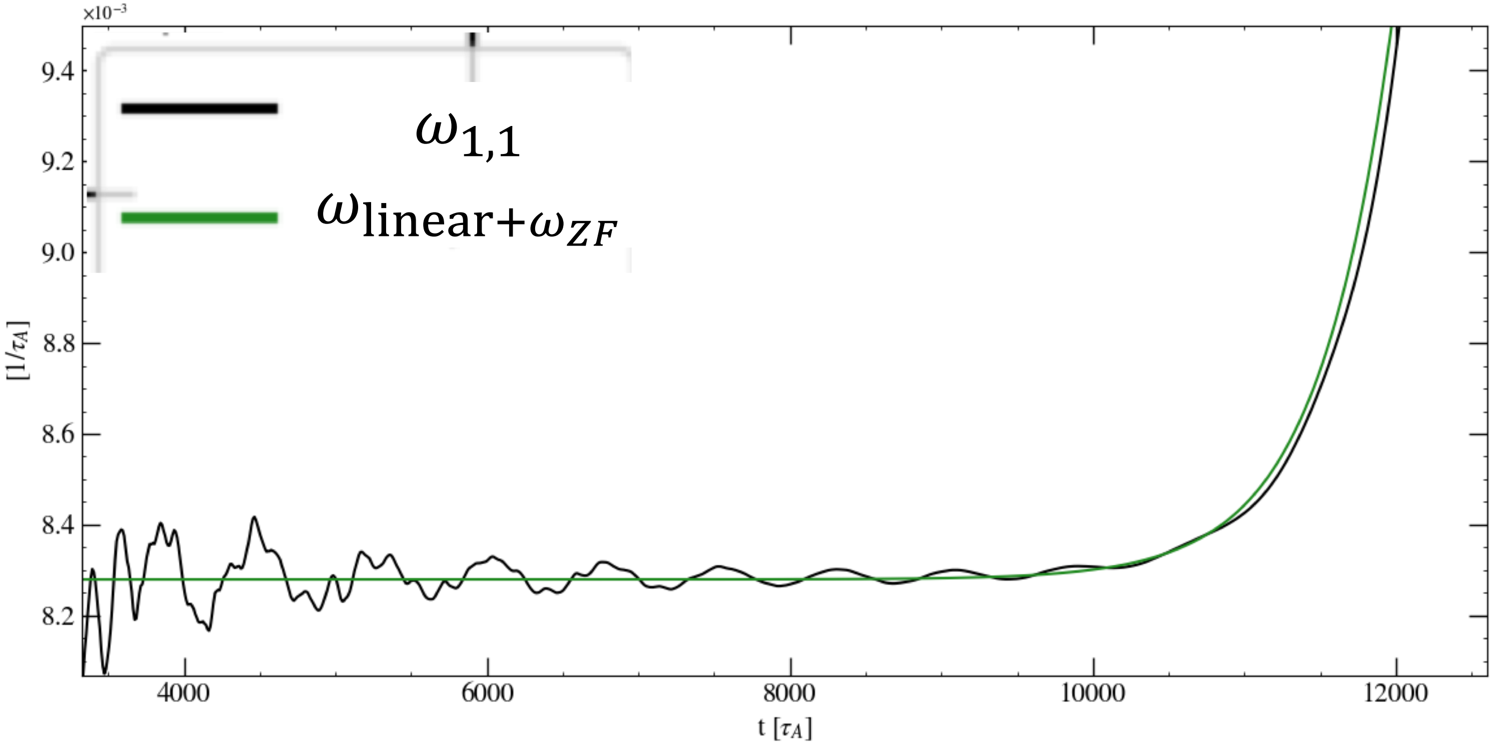


Kinetic : nonlinear, MHD : nonlinear

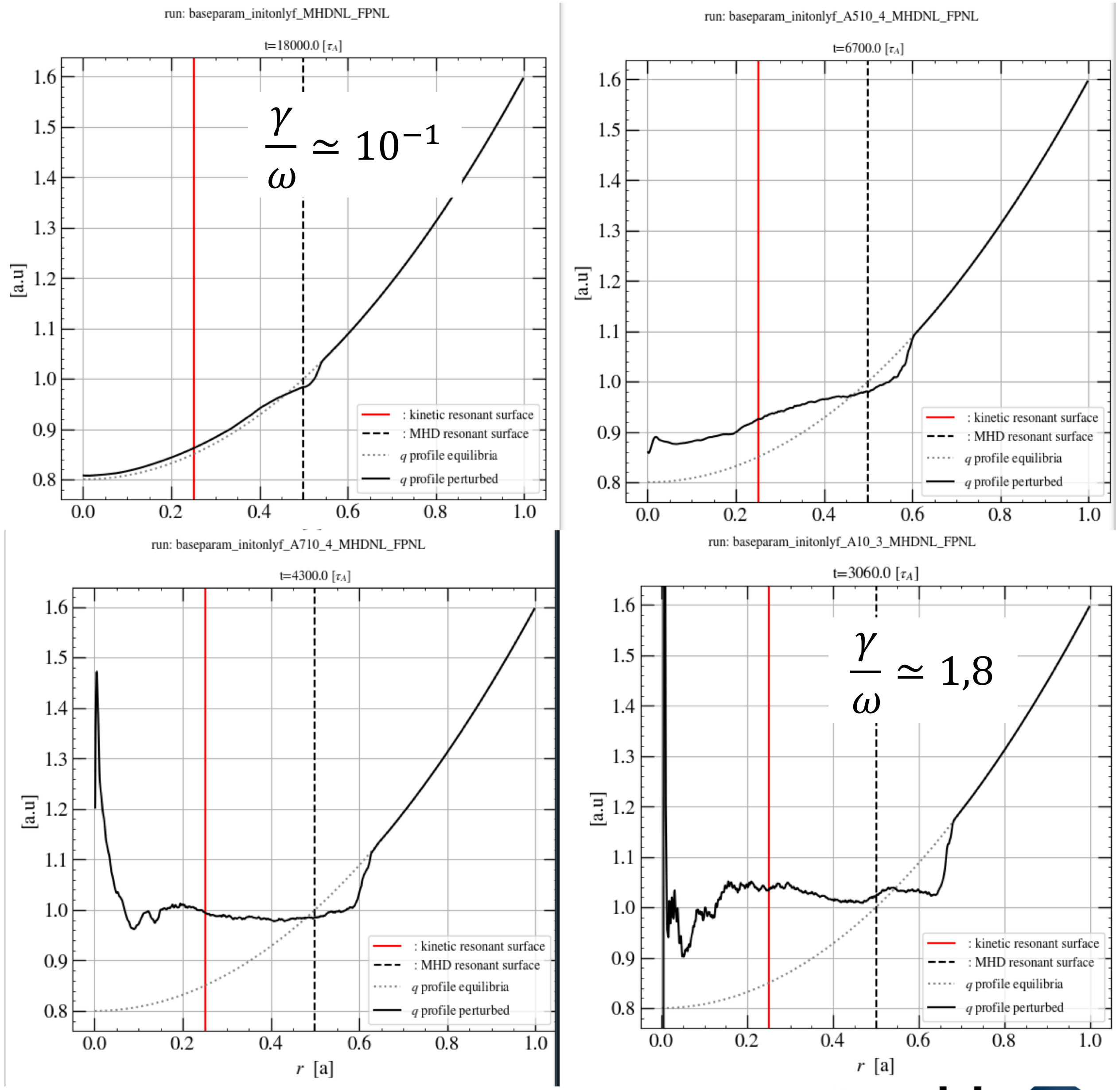


Zonal flow impact for MHD nonlinear effects only for $\frac{\gamma}{\omega} \simeq 10^{-1}$

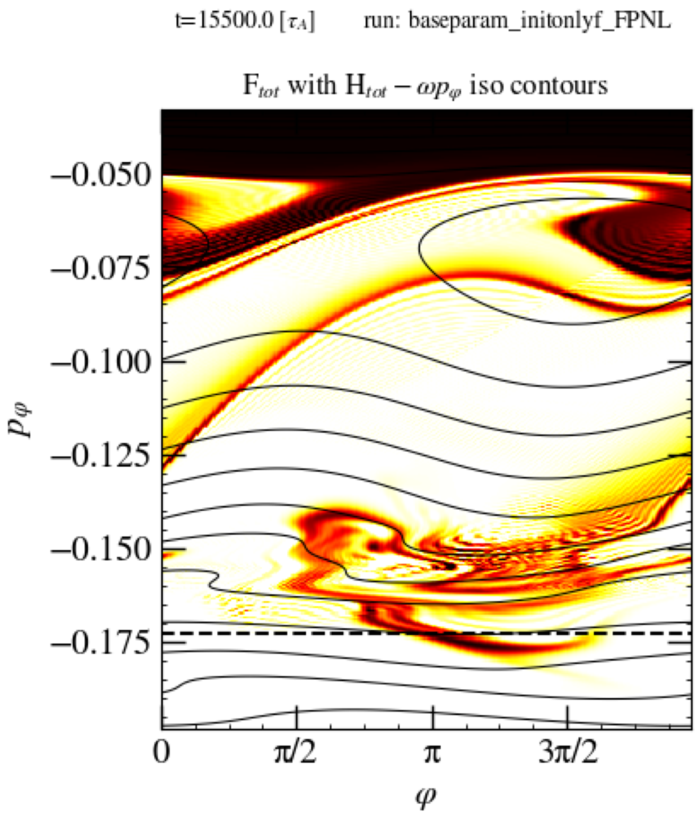
[Odblom, et al., 2002]

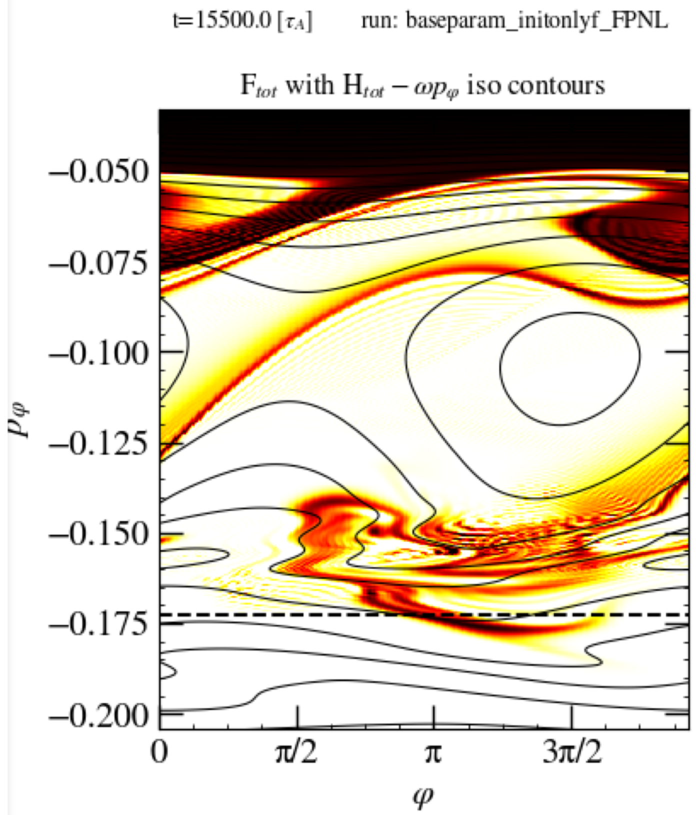


q profile nonlinear evolution



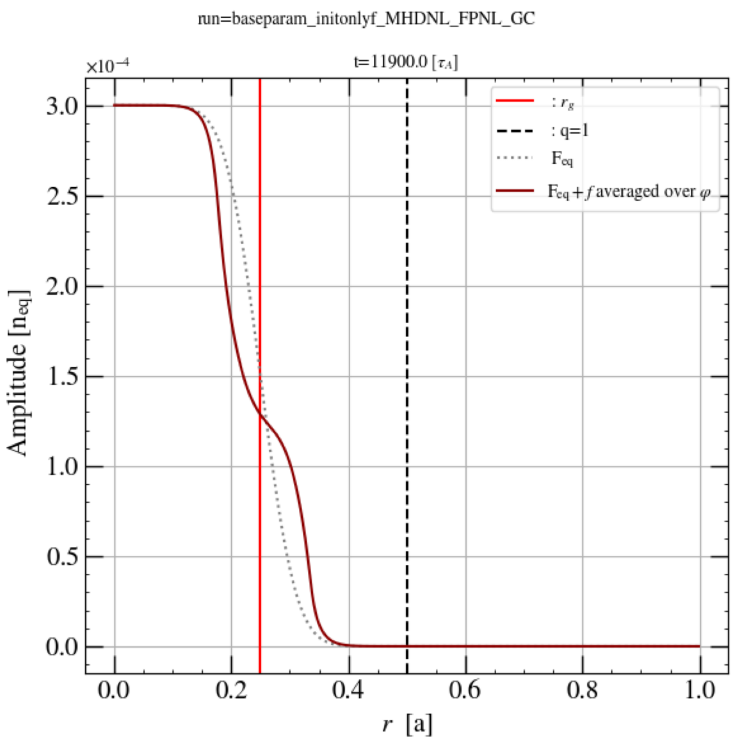
Possible chaos for $\frac{\gamma}{\omega} \simeq 10^{-1}$

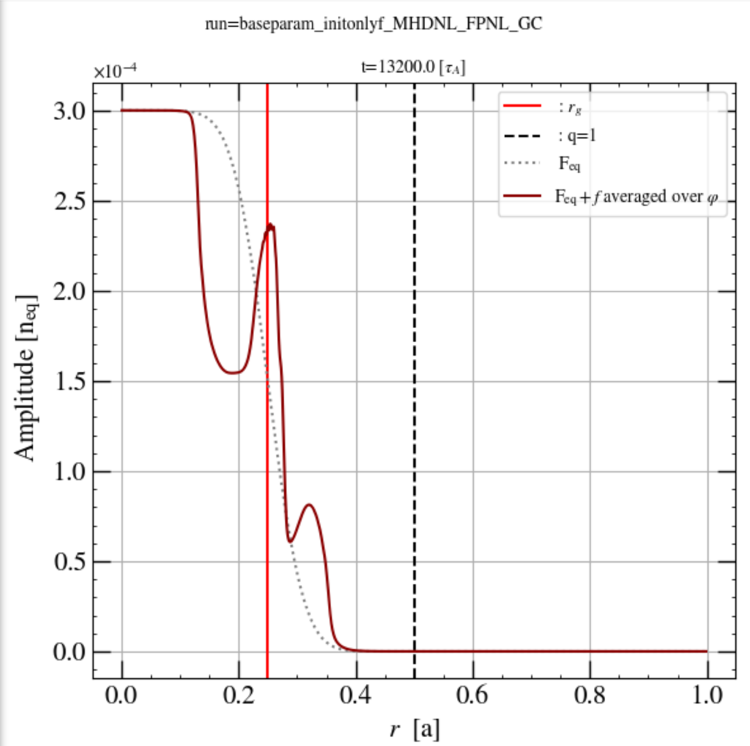


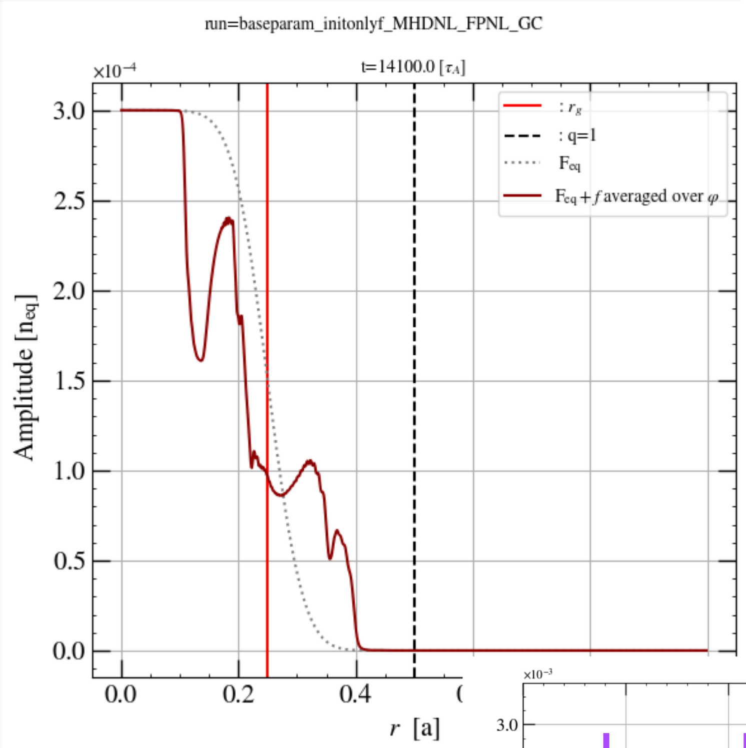


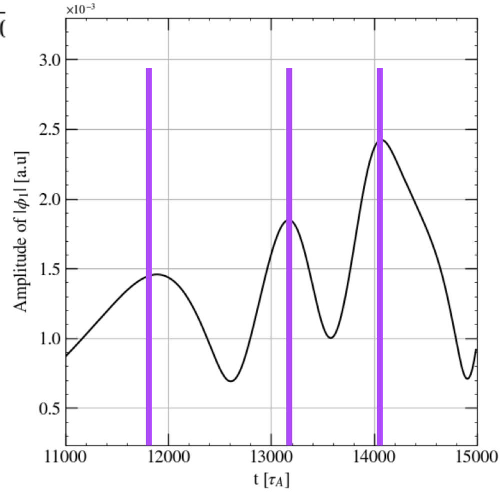
Full modes coupling 1/2

$$\frac{\gamma}{\omega} \simeq 10^{-1}$$



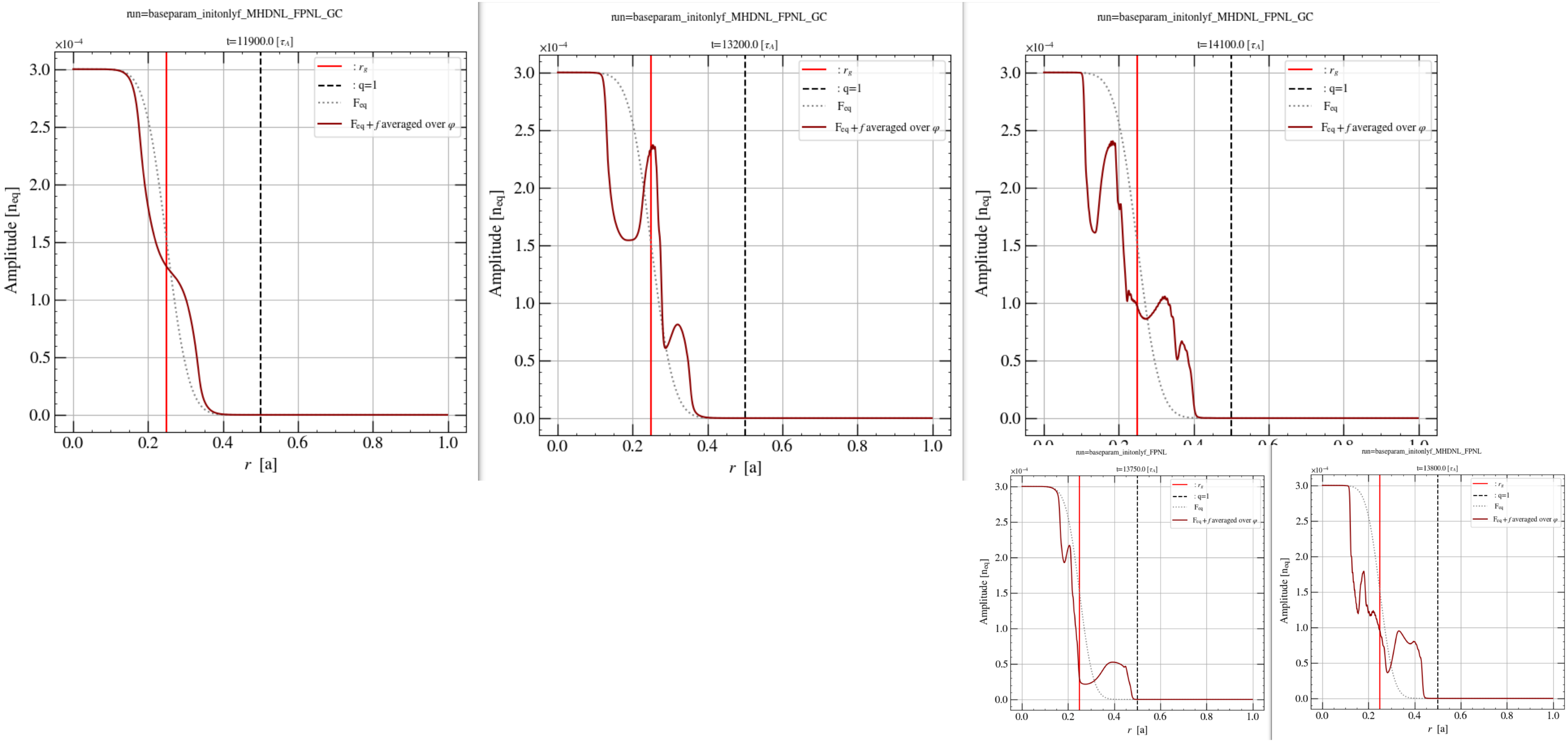






Full modes coupling 1/2

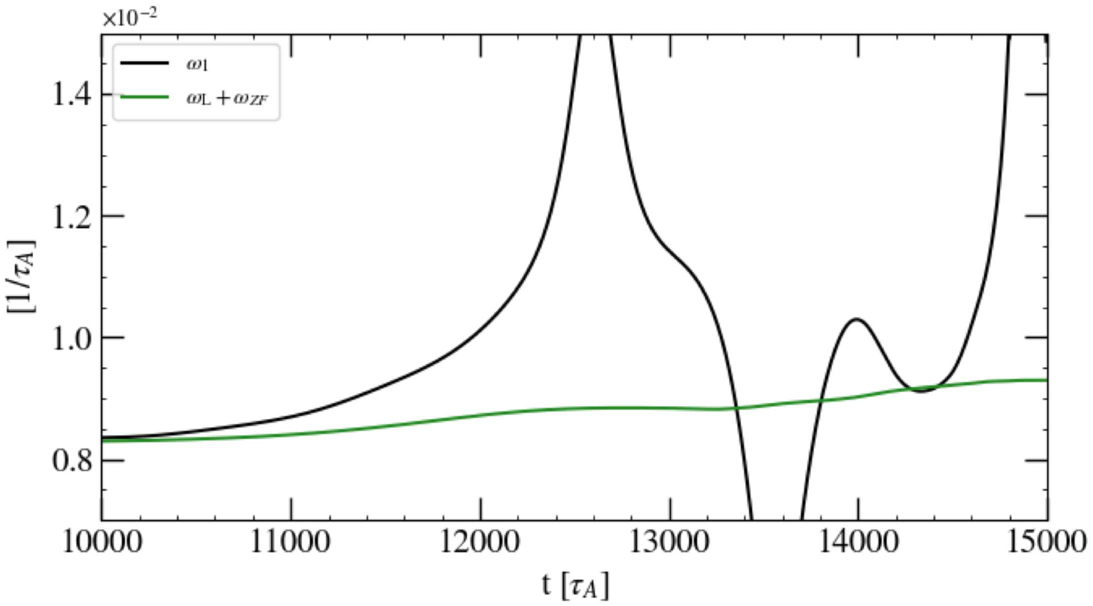
$$\frac{\gamma}{\omega} \simeq 10^{-1}$$



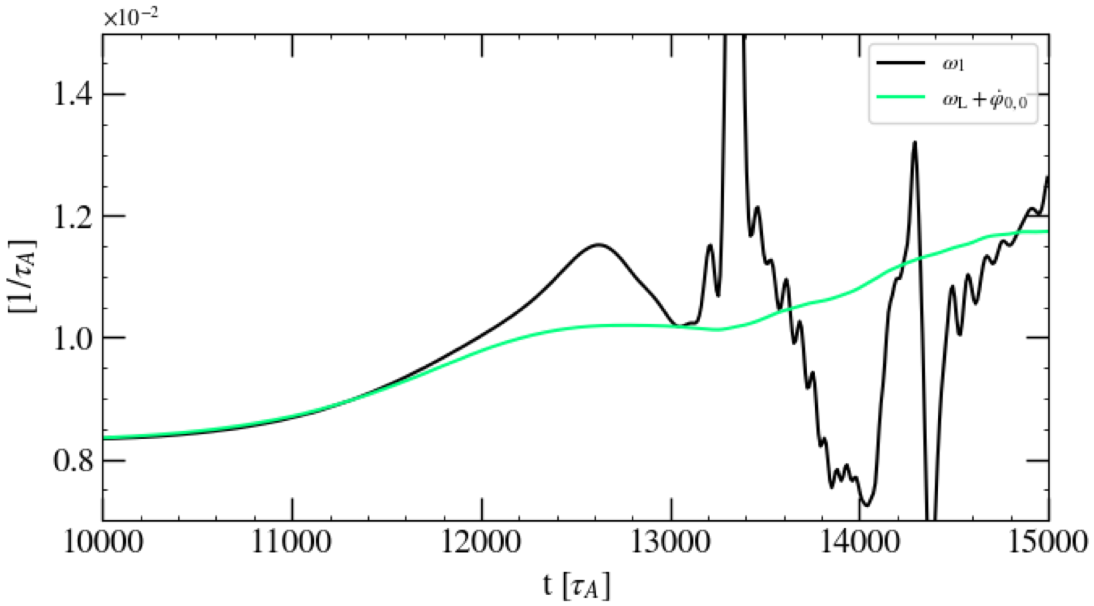
Full modes coupling 2/2

$$\frac{\gamma}{\omega} \simeq 10^{-1}$$

Measured on $\phi_{1,1}$



Measured on $f_{1,1}$



$$\omega_{ZF} = -\frac{1}{\mathrm{a}}\frac{1}{\mathrm{B}}\frac{\partial\tilde{\phi}_{0,0}}{\partial r}$$

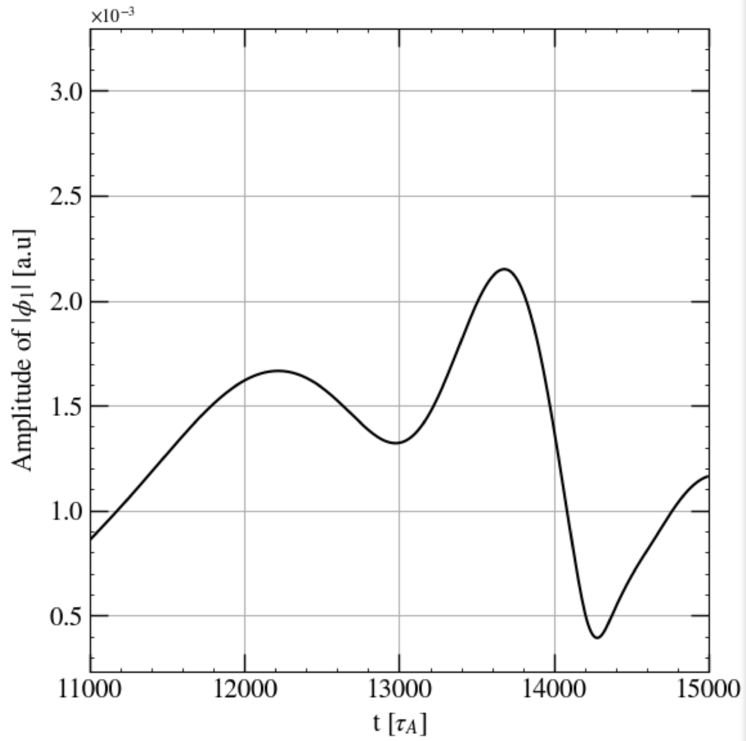
$$\dot{\varphi}_{0,0} = \frac{\partial\tilde{H}_{0,0}}{\partial p_\varphi} = -\frac{1}{eR_0\left(\frac{\partial\Psi}{\partial r}\right)}\frac{\partial e\tilde{\phi}_{0,0}}{\partial r} = -\frac{1}{R_0\left(\frac{\partial\Psi}{\partial r}\right)}\frac{\partial\tilde{\phi}_{0,0}}{\partial r}$$

$$\omega_{ZF} < \dot{\varphi}_{0,0}$$

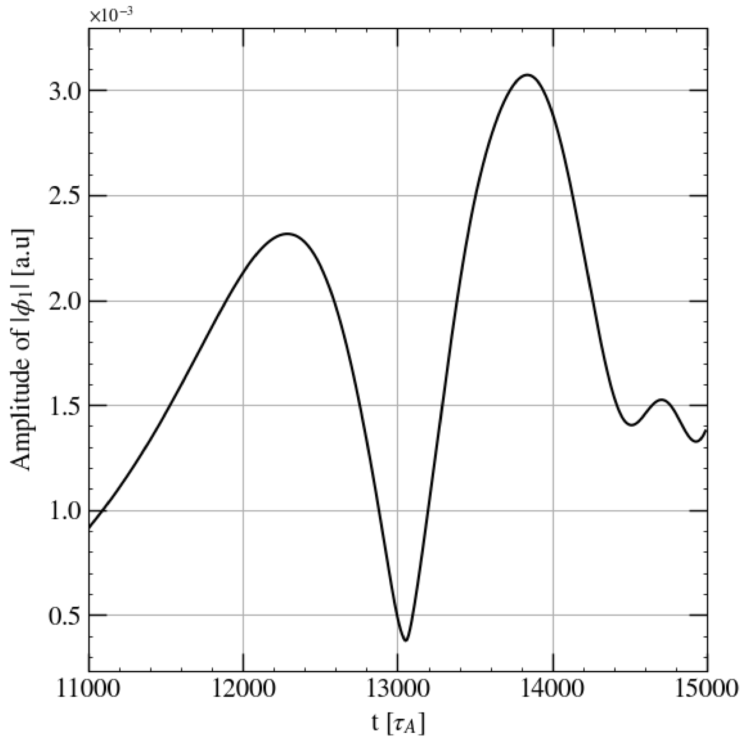
Energy exchange 1/2

$$\frac{\gamma}{\omega} \simeq 10^{-1}$$

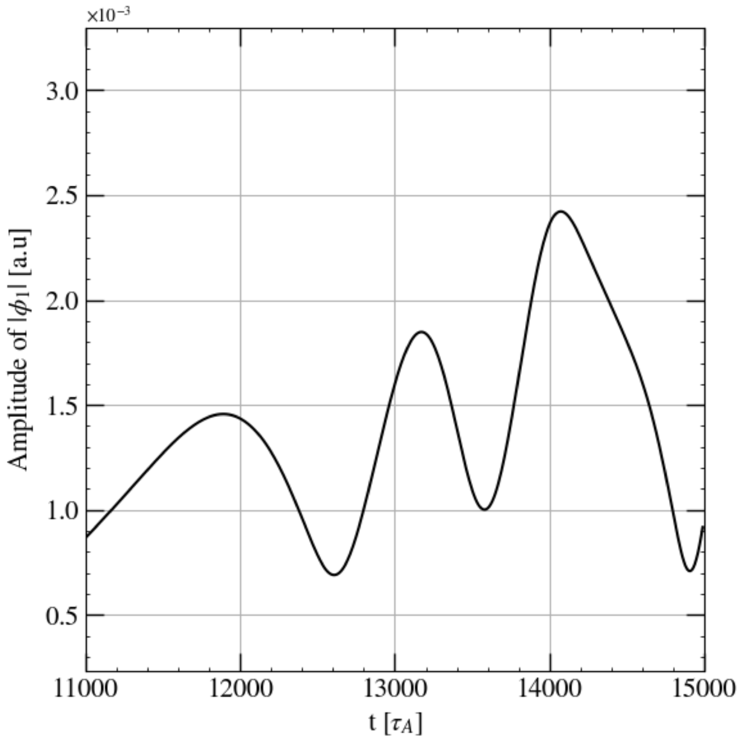
Kinetic : nonlinear, MHD : linear



Kinetic : nonlinear, MHD : nonlinear



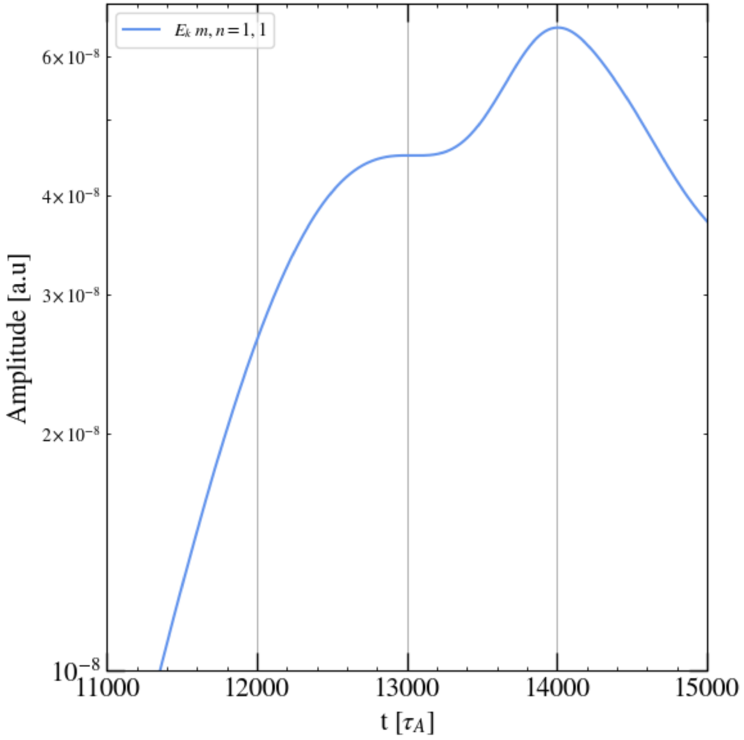
Kinetic : nonlinear, MHD : nonlinear
Full modes coupling



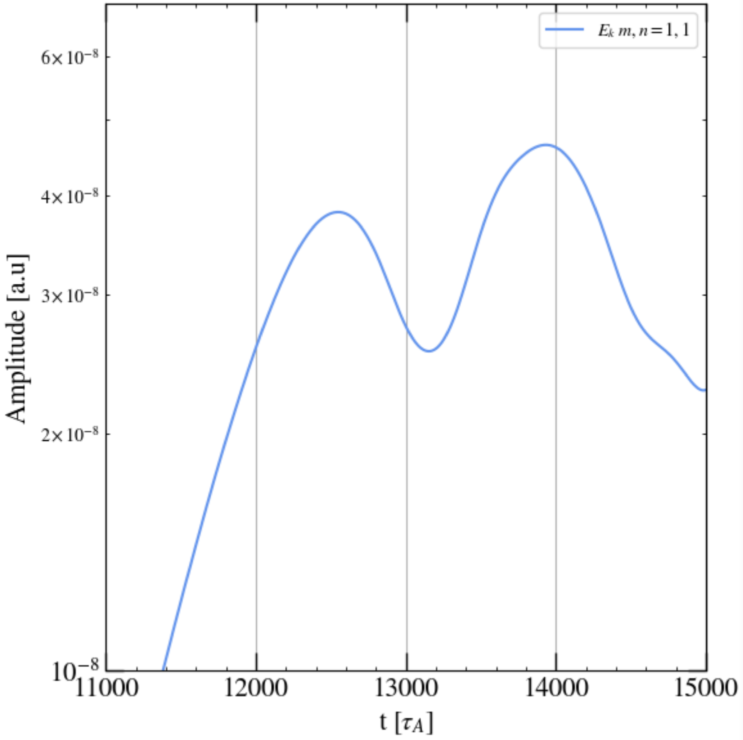
Energy exchange 2/2

$$\frac{\gamma}{\omega} \simeq 10^{-1}$$

Kinetic : nonlinear, MHD : linear



Kinetic : nonlinear, MHD : nonlinear



Kinetic : nonlinear, MHD : nonlinear
Full modes coupling