Energetic spectra of relativistic reconnection at large scales using implicit PIC methods.

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Energy spectra are important in astrophysics

Observations imply energetic particle spectra

-Observations of neutrinos and cosmic rays from active galactic nuclei (AGN) e.g. NGCI068 and NGC7469

Detected by IceCube Collaboration between June 2019 and October 2023 Sommani et al. (**2024**) arXiv: 2403.03752, Cotton et al. (**2008**)

-requires an acceleration mechanism -Could be produced by magnetic reconnection.

Sironi et al. (2014), Guo et al. (2016), Werner et al. (2015)

-Huge system sizes difficult to simulate.











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A strongly relativistic regime











What is a relativistic plasma?

Relativistic plasma

-Relativistic Alfvén speed (high magnetization)

$$\sigma_{\alpha,c} \equiv \frac{B^2}{4\pi n m_{\alpha} c^2} > 1$$

-Relativistic temperatures

$$\frac{T_{\alpha}}{m_{\alpha}c^2} > 1$$

-At times, electron-positron (pair) dominant plasma.



High magnetization \rightarrow free energy for energetic spectra







Outline

Introduction

Tearing instability for pairs Benchmark from explicit PIC simulations

Reconnection at large scales Advantages of semi-implicit methods









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Let's consider a Harris equilibrium

Harris equilibrium

$$B_{x} = B_{0} \tanh\left(\frac{y}{\lambda}\right)$$
$$n = n_{0} \operatorname{sech}^{2}\left(\frac{y}{\lambda}\right)$$

Constant and uniform v_d and T

Not only in force balance

$$\frac{B_0^2}{8\pi} = 2n_0 T = n_0 \left(T_e + T_i \right)$$

Is in kinetic equilibrium









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Zelenyi model corrected for relativistic drifts

Non-relativistic :
$$\frac{v_T}{c} \ll 1$$

$$\frac{\gamma a}{v_T} = C_1(ka) \frac{1}{\Gamma_d^{5/2}} \left(\frac{u_d}{v_T}\right)^{3/2}$$

$$\frac{u_d}{v_T} = \frac{\rho_{L,C}}{a} = \frac{\sqrt{\Gamma_D} d_{e,C}}{a}$$

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From Zelenyi 1979 + Hoshino 2020









OSIRIS framework

Massively Parallel, Fully Relativistic • Particle-in-Cell Code

- Support for advanced CPU / GPU architectures ٠
- Extended physics/simulation models ٠
- AI/ML surrogate models and data-driven discovery •

Open-source version available

Open-access model

- 40+ research groups worldwide are using OSIRIS
- 400+ publications in leading scientific journals
- Large developer and user community
- Detailed documentation and sample inputs files available
- Support for education and training

Using OSIRIS 4.0

- The code can be used freely by research institutions after signing an MoU Open-source version at:
 - https://osiris-code.github.io/



Ricardo Fonseca: ricardo.fonseca@tecnico.ulisboa.pt



Linear grows at predicted wavelength

ka











Tearing modes consist of ...

- -Heating (island centers)
- -Flows away from x-line
- -Density peaked at island centers
- -Current also peaked at island centers



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Growth rates are measured from transverse field



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Simulations match Zelenyi (non-relativistic)

From Zelenyi 1979

$$\frac{\gamma a}{v_T} = C_1(ka) \frac{1}{\Gamma_d^{5/2}} \left(\frac{\rho_{L,C}}{a}\right)^{3/2}$$

Non-relativistic :
$$\frac{v_T}{c} \ll 1$$

Good fit, holding

$$a$$
constant
 $\rho_{L,C}$

 \mathcal{A} = 20 \mathcal{A}

and

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Simulations match Zelenyi (relativistic)

From Zelenyi 1979 + Hoshino 2020

Non-relativistic

$$\frac{\gamma a}{v_T} = C_1(ka) \frac{1}{\Gamma_d^{5/2}} \left(\frac{u_d}{v_T}\right)^{3/2}$$

Relativistic

$$\frac{\gamma a}{c} = C_2(ka) \frac{1}{\Gamma_d^{5/2}} \left(\frac{u_d}{c}\right)^{3/2}$$

$$\frac{Good \text{ fit, holding}}{\rho_{L,R}} = \frac{u_d}{c} \text{ constant}$$

ya/c





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Simulations match Zelenyi (relativistic)











Tearing for pair-plasmas

Prediction follow model

-Zelenyi model matches PIC simulations

Zelenyi and Krasnosel'skikh (1979)

-Extrapolations to large $\stackrel{a}{-\!\!-\!\!-\!\!-}\gg 1$ can $ho_{L,R}$ give estimate for tearing rate in thick current sheets.

-Electron-positron (pair) dominant plasma assumed, but growth rate is modified by only a factor ~ 1 .

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Estimated formation times for astrophysical objects



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Critical thickness before tearing



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Energetic spectra found in explicit PIC simulations



Guo et al. (**2016**)

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Energetic spectra found in explicit PIC simulations



Guo et al. (**2016**)

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What is an explicit particle-in-cell (PIC) code?



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 $u_{i,n+1}$ and E_{n+1} are based on the fields E_n and momenta $u_{i,n}$ from the previous time step

Leads to instabilities if time steps and spaces are not fully resolved









What is an implicit particle-in-cell (PIC) code?



$$\frac{\partial u_i}{\partial t} = \frac{q}{mc} \left(E + \frac{1}{2} \int_{i}^{\infty} \frac{1}{2} \int_{i}^{\infty} \frac{q_i n_i v_i}{q_i n_i v_i} \right)$$
$$\frac{\partial B}{\partial t} = -c \nabla \times E$$
$$\frac{\partial E}{\partial t} = c \nabla \times B - 4$$

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 $\frac{v_i \times B}{c}$

 $v_i = \frac{u_i}{\gamma_i}$

All equations are solved selfconsistently in terms of $u_{i,n}$, E_n , $u_{i,n+1}$, and E_{n+1} usually by iteration

Avoids instabilities, allowing for under-resolution of time and space

 $4\pi j$









What is an semi-implicit particle-in-cell (PIC) code?



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Avoids instabilities, allowing for under-resolution of time and space









Must resolve in explicit codes



$$\lambda_D^2 = \frac{T}{4\pi m n e^2} \qquad d_e^2 = \frac{m_e c^2}{4\pi m n e^2} \qquad d_i^2 = \frac{m_i c^2}{4\pi m n e^2}$$

$$\lambda_D = \frac{v_T}{c} d_e = \frac{v_T}{c} \sqrt{\frac{m_e}{m_i}} d_i$$









We can under-resolve in implicit/ semi-implicit codes



If we are mainly interested in ion scales, we can under-resolve the electron scales



$$\lambda_D^2 = \frac{T}{4\pi m n e^2} \qquad d_e^2 = \frac{m_e c^2}{4\pi m n e^2}$$

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semi-implicit schemes tend to be $\sim 10 \times$ slower

But,

Factor of $\sqrt{m_i/m_e}c/v_T$

faster for each space direction

Most benefit in 3D







We can under-resolve in implicit/ semi-implicit codes



If we are mainly interested in ion scales, we can under-resolve the electron scales



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Relativistic Semi-Implicit code ReISIM

Bacchini 2023 [https://doi.org/10.48550/arXiv.2306.04685]

ReISIM is EcSIM (Lapenta 2017): modified such that it can do simulations of relativistic plasmas.

Tested relativistic two-stream



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Tested relativistic shock evolution







Setup to simulate tearing

y/p_





 X/ρ_L





No background

$$n_b = 0$$

Double Harris Sheet configuration

$$\frac{d}{d_{2}} = \frac{T_{i,H}}{m_{e}c^{2}} = 10 \qquad \frac{a}{d_{i}} = 8.94 \qquad n = \frac{n_{0}}{2}\operatorname{sech}^{2}\left(\frac{y \mp L_{y}/2}{a}\right)$$
$$\frac{d}{d_{i}} = 0.05 \qquad B_{x} = B_{0}\left[1 - \tanh\left(\frac{y - L_{y}/2}{a}\right) + \tanh\left(\frac{y + L_{y}/2}{a}\right)\right]$$







Tearing fits theory with ReISIM and OSIRIS

Explicit OSIRIS simulation



Schoeffler et al. (2025) in review



Implicit ReISIM simulation







One can get away with less resolution with ReISIM



Schoeffler et al. (2025) in review

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Energy conservation depends on time resolution

Fits theory with enough particles per cell





One can get away with less resolution with RelSIM



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Setup to simulate nonlinear reconnection



 X/ρ_L

y/p_

 $\frac{T_{e,H}}{m_e c^2}$ u_d

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Background parameters $\frac{T_i}{T_e} = 1$ $\sigma_{i,c} = 100$ $\frac{T_{e,b}}{m_e c^2} = 10 \qquad \qquad \frac{m_i}{m_e} = 100$

Double Harris Sheet configuration

$$= 10 \qquad \frac{a}{d_i} = 0.5 \qquad \qquad n = \frac{n_0}{2} \operatorname{sech}^2 \left(\frac{y \mp L_y/2}{a}\right)$$
$$= 0.4 \qquad \frac{n_0}{n_b} = 269 \qquad \qquad B_x = B_0 \left[1 - \tanh\left(\frac{y - L_y/2}{a}\right) + \tanh\left(\frac{y + L_y/2}{a}\right)\right]$$







Previous results reproduced with OSIRIS









Explicit results reproduced with implicit method



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$$\frac{c}{\omega_{pe}} = \sqrt{\frac{m_e c^2}{4\pi n_b e^2}}$$
$$\frac{c}{\Omega_c} = \frac{m_e c^2}{eB_0}$$
$$\gamma_T \rho_0 = \frac{2T}{m_e c} \rho_0$$

$$l_e = 1.86\rho_0 = 0.093\rho_{Le,R}$$







Magnetization dependence of spectra confirmed

y/d_e

u/n

Magnetization σ







 $t\omega_{pe} = 1110.000$



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System size dependence of spectra confirmed

y/d_e

System size L/d_e









Schoeffler et al. (2025) in review

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Fluid codes are another strategy

Simulation done with relativistic 5-moment fluid simulation MuPhyll, with test particles.

2-fluid equations:

$$\partial_t \begin{pmatrix} \gamma_s \rho_s \\ \frac{w_s}{c^2} \gamma_s \mathbf{u}_s \\ w_s \gamma_s^2 - p_s \end{pmatrix} + \nabla \cdot \begin{pmatrix} \rho_s \mathbf{u}_s \\ \frac{w_s}{c^2} \mathbf{u}_s \otimes \mathbf{u}_s + p_s \mathbb{1} \\ w_s \gamma_s \mathbf{u}_s \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mu_s \gamma_s \rho_s (\mathbf{E} + \frac{\mathbf{u}_s}{\gamma_s} \times \mathbf{B}) \\ \mu_s \rho_s \mathbf{u}_s \cdot \mathbf{E} \end{pmatrix}$$

Maxwell:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \,\epsilon_0 \partial_t \mathbf{E}, \quad \nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad \nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho_c, \quad \nabla \cdot \mathbf{B} = 0$$
$$\rho_c = \sum_s \mu_s \, \gamma_s \, \rho_s, \quad \mathbf{j} = \sum_s \mu_s \, \rho_s \, \mathbf{u}_s$$

Simulation parameters

$$\sigma_{e,c} = 30 \qquad \qquad \frac{u_d}{c} = 0.6$$
$$L = 2048 \,\rho_0 \approx 68 \,\sigma\rho_0 \qquad \qquad \frac{n_0}{n_b} = 10$$

Parameters from Werner et al. (2015)

Wilbert et al. (in preparation)











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$$\rho_c = \sum_s \mu_s \, \gamma_s \, \rho_s, \quad \mathbf{j} = \sum_s \mu_s \, \rho_s \, \mathbf{u}_s$$

Simulation parameters









Wilbert et al. (in preparation)





3D simulations with ReISIM













Conclusions

Tearing studied using explicit PIC simulations

- Zelenyi theory along with Hoshino correction is confirmed with OSIRIS
- A new benchmark for tearing simulations available
- Large scale current sheets driven by tearing ruled out

Semi-implicit model has strong computational advantage for tearing

- Smaller spatial resolution is needed
- Lower time resolution and particles per cell for the same energy conservation
- Computational savings as high as 256 x

Large scale Relativistic reconnection

- Can be modelled with semi-implicit PIC (or even fluid) simulations
- We can now explore larger 2D and 3D system sizes with kinetic methods
- Energetic spectra may help explain cosmic rays and neutrinos found near AGNs











