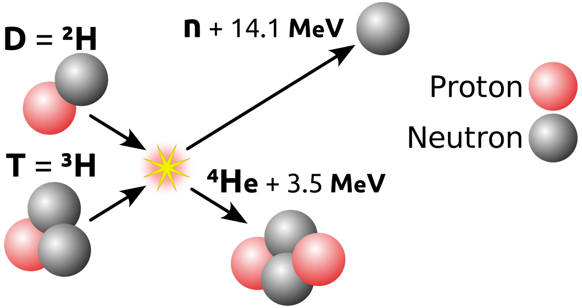
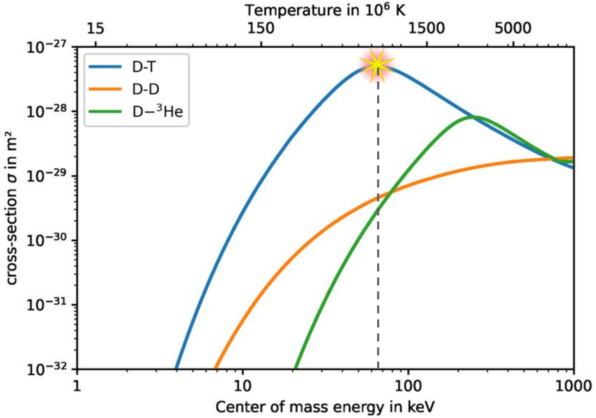
Investigation of magnetic island growth and saturation mechanisms in the gyrokinetic framework of GYSELA

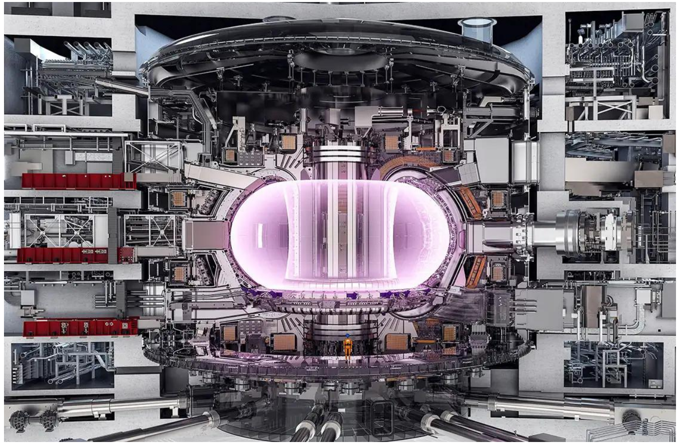
Roméo Bigué, *Magali Muraglia*, *Peter Donnel*, *Yanick Sarazin*, *Xavier Garbet*

Taming fusion energy

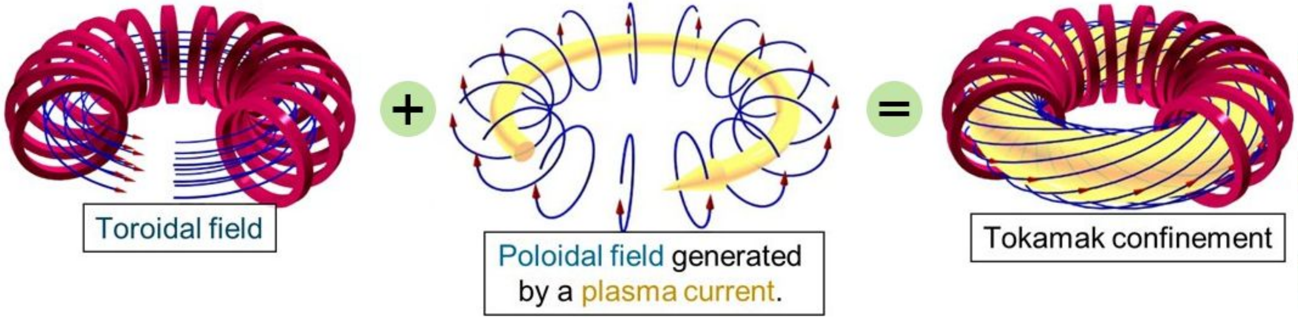


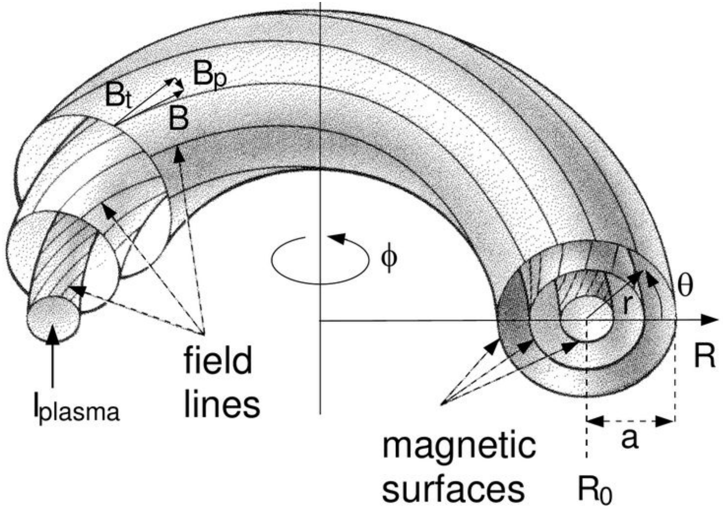


Need to control hot and dense plasma



Tokamak configuration

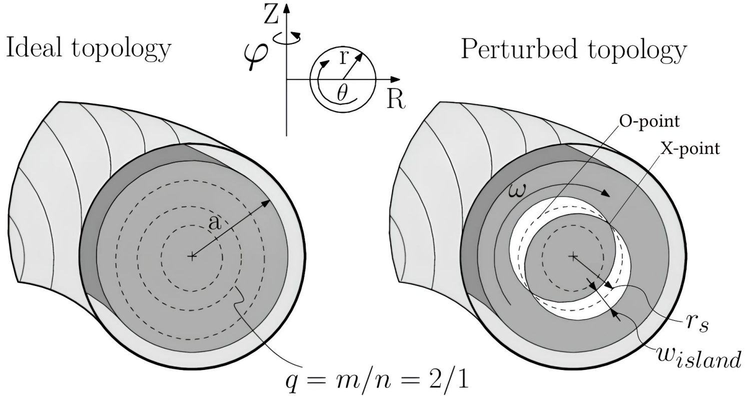




- Ideally : particles confined on nested flux surfaces
- helicity of the field lines : $q = \dfrac{rB_t}{RB_p} = \dfrac{\#\text{toroidal}}{\#\text{poloidal}}$

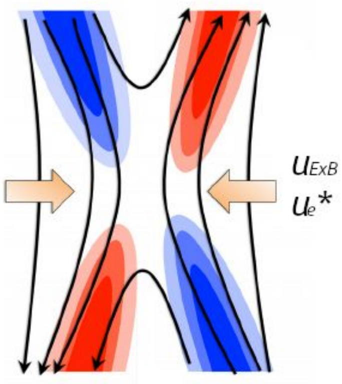
Magnetic island

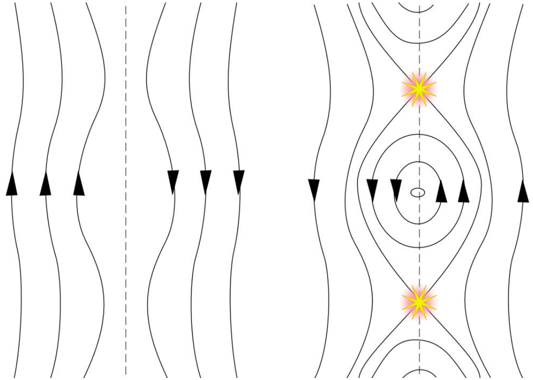
$$\tilde{A}_{\|}(r,\theta,\varphi) = \sum_{m,n} A_{\| mn}(r) e^{im\theta + in\varphi} + \text{c.c}$$



Current gradient can drive **tearing instability**

Large scales : plasma and **B** are frozen-in
Small scales : non-ideal phenomena (ex : **η**,**d_e**) can induce magnetic reconnection



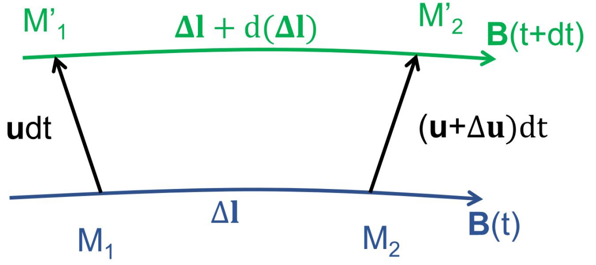


Non-ideal phenomena and magnetic reconnection

Ideal MHD rules at large scale

$$\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = 0$$

Magnetic connectivity : $\boldsymbol{\Delta l} \times \boldsymbol{B}$



Magnetic connectivity is conserved at large scales

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\Delta l} \times \boldsymbol{B}) = [\boldsymbol{\nabla} \times (\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})] \times \boldsymbol{\Delta l} = 0$$

Non-ideal phenomena and magnetic reconnection

Non-ideal MHD models

$$\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B} = \boldsymbol{E}_{\mathbf{res}} + \boldsymbol{E}_{\mathbf{therm}} + \boldsymbol{E}_{\mathbf{iner}} = \boldsymbol{E}^{\mathbf{NI}}$$

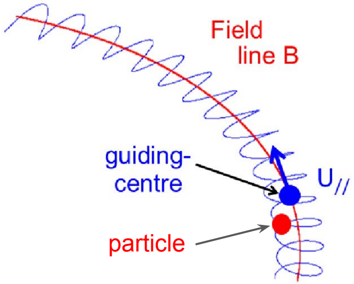
Some terms are **allowing** reconnection

$$\frac{\mathrm{d}}{\mathrm{d}t}(\boldsymbol{\Delta l} \times \boldsymbol{B}) = [\boldsymbol{\nabla} \times E_{\parallel}^{\mathrm{NI}} \boldsymbol{b}] \times \boldsymbol{\Delta l}$$

$$\boldsymbol{E}_{\mathbf{res}} = \eta \boldsymbol{j} \qquad \boldsymbol{E}_{\mathbf{therm}} = \frac{1}{n_e q_e} \boldsymbol{\nabla} \cdot \overline{\overline{\boldsymbol{P}_e}} \qquad \boldsymbol{E}_{\mathbf{iner}} = \frac{m_e}{n_e e^2} \left(\partial_t \boldsymbol{j} + \boldsymbol{\nabla} \cdot \left(\overline{\overline{\boldsymbol{uj}}} + \overline{\overline{\boldsymbol{ju}}} - (1/n_e e) \overline{\overline{\boldsymbol{jj}}} \right) \right)$$

Fluid models hold key ingredients

Why bother with a gyrokinetics description ?



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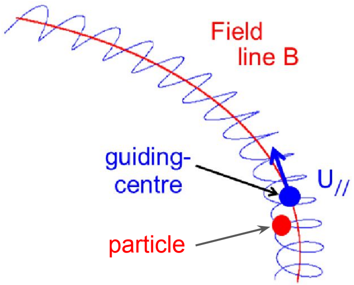
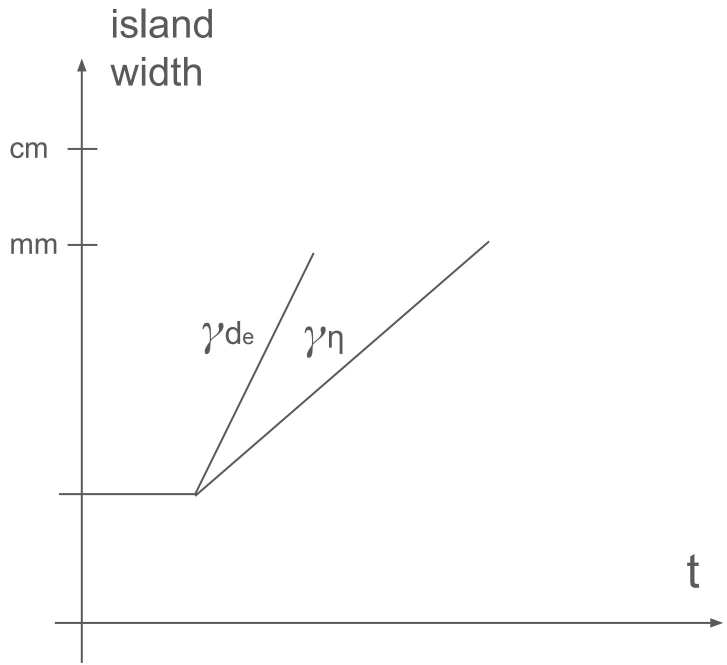


Electromagnetic gyrokinetics opens the way :

- more refined non-ideal effects description :
  - η : collision operator
  - d_e : kinetic evolution
- toroidal turbulent transport modelling

Magnetic island growth process

sketch for illustrative purpose



- Resistive & inertial tearing linear instability

Magnetic island growth process

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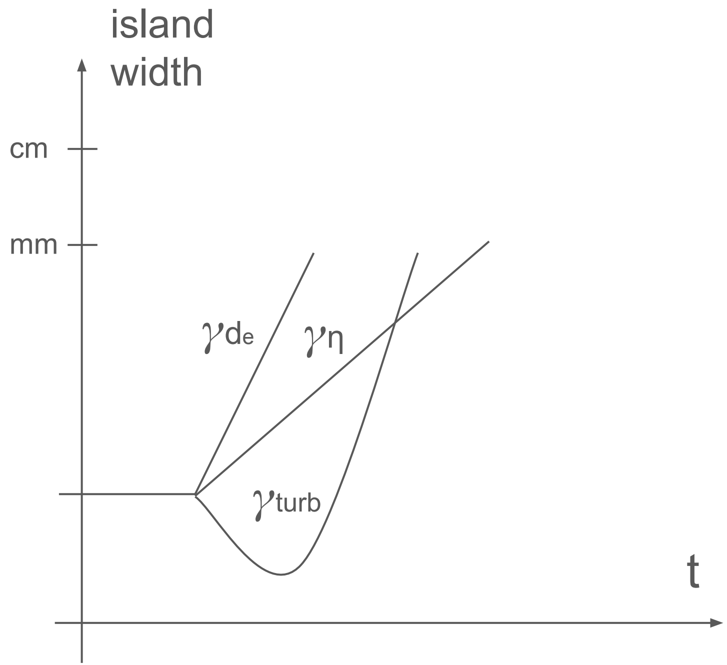
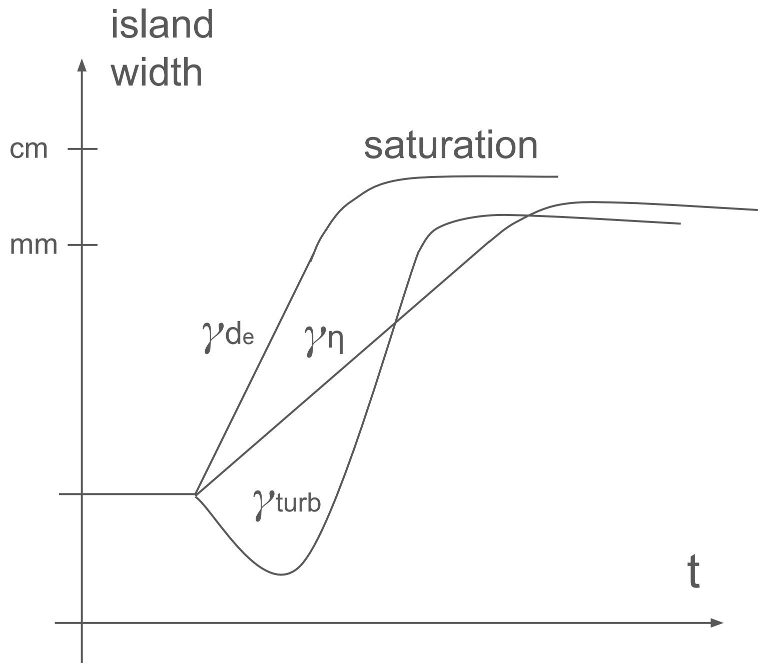


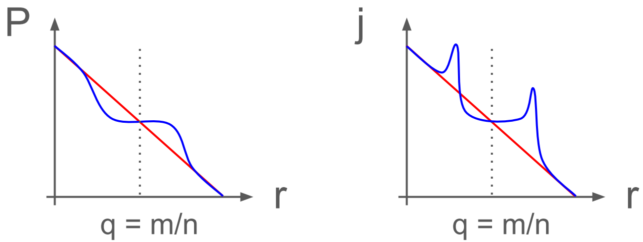
- Resistive & inertial tearing linear instability
- Turbulence can force magnetic reconnection

[Waelbroeck 2008, Hornsby 2010]

Magnetic island growth process

sketch for illustrative purpose





- Resistive & inertial tearing linear instability
- Turbulence can force magnetic reconnection [Waelbroeck 2008, Muraglia 2011, Hornsby 2015]
- NL coupling with equilibrium P, j flattening
- Prediction of saturated island width ?

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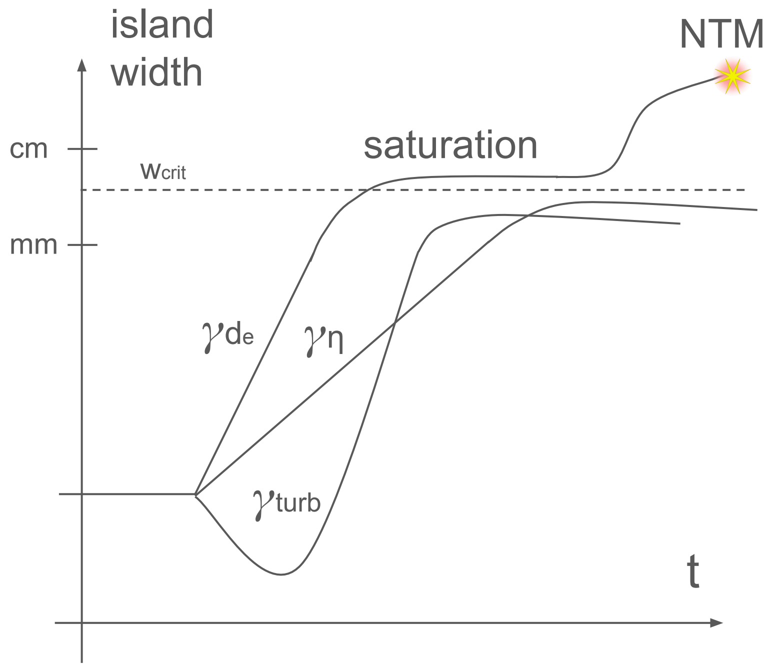

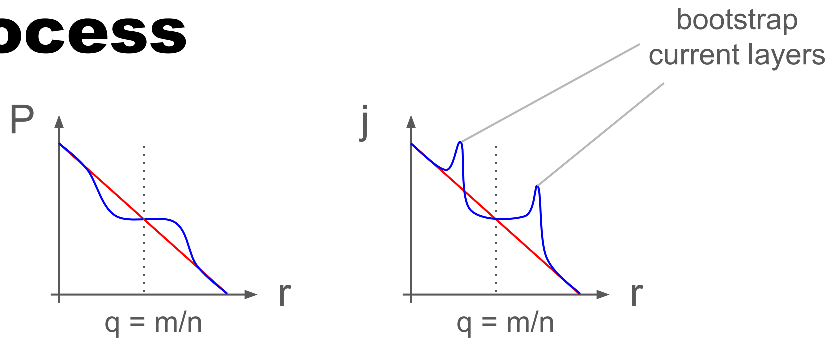

- Resistive & inertial tearing linear instability
- Turbulence can force magnetic reconnection [Waelbroeck 2008, Muraglia 2011, Hornsby 2015]
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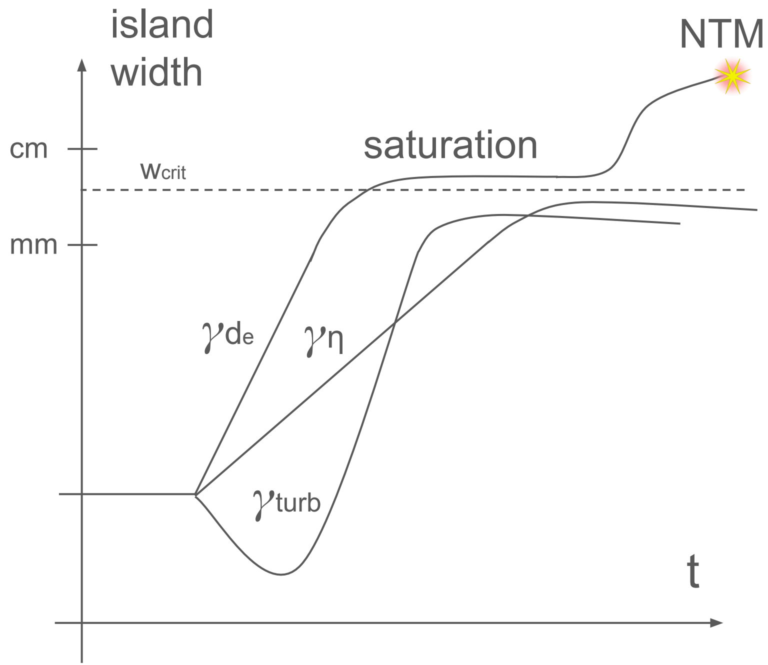
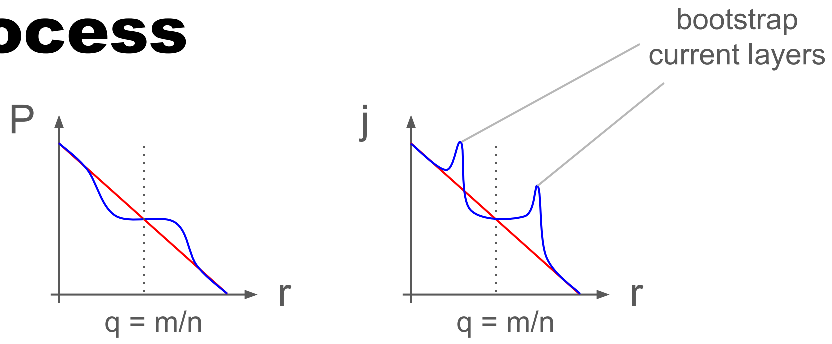
- Bootstrap current layers can drive NTM [Chang 1995]
- NTM can cause plasma disruptions

Magnetic island growth process

sketch for illustrative purpose





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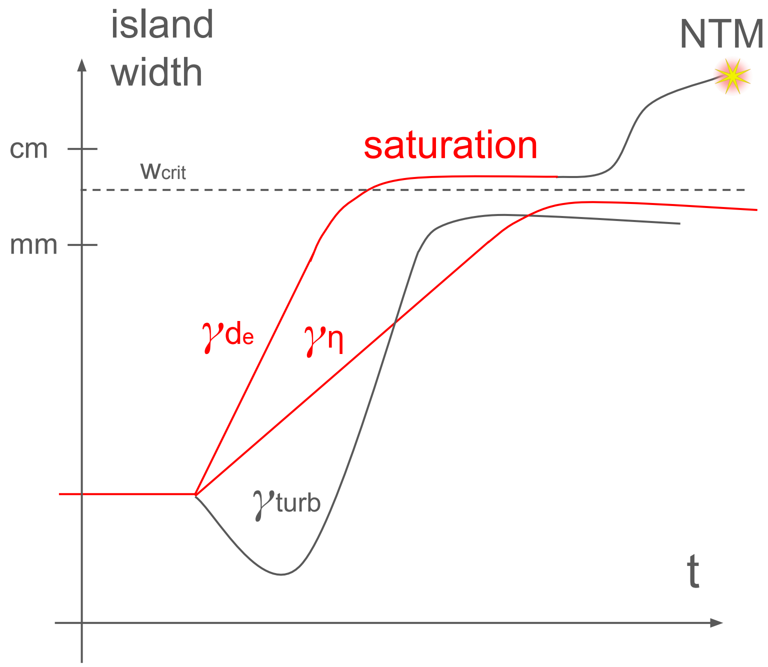
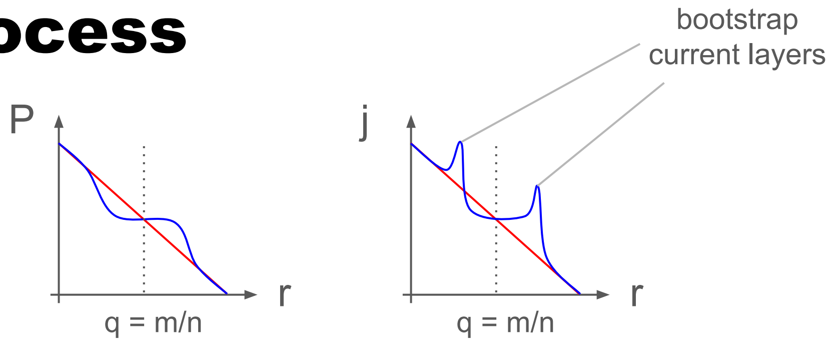
MR in tokamak :

mechanisms - drive - saturation - detection - control

Magnetic island growth process

sketch for illustrative purpose





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Our goal : self-consistent saturation of linearly unstable η , d_e tearing modes using GYSELA

Electromagnetic GYSELA

Ampère equation

Quasi-neutrality equation

$\overline{F}_s(r, \theta, \varphi, v_{\|}, \mu)$

$\phi(r, \theta, \varphi)$
$A_{\|}(r, \theta, \varphi)$

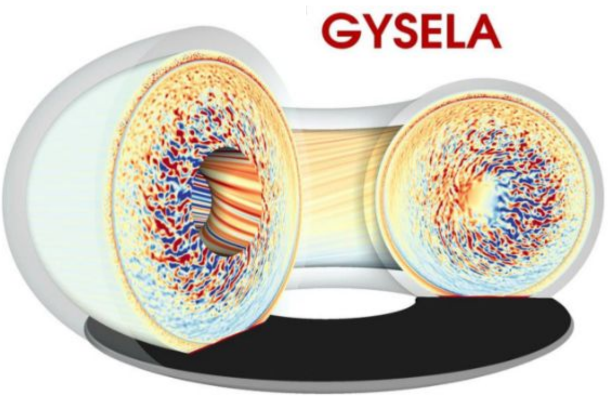
Kinetic plasma response

gyrokinetic Boltzmann 5D equations

[Grandgirard 2016, Gillot 2020]

gyrokinetic **se**mi-**la**grangian code

global , full-f , flux-driven



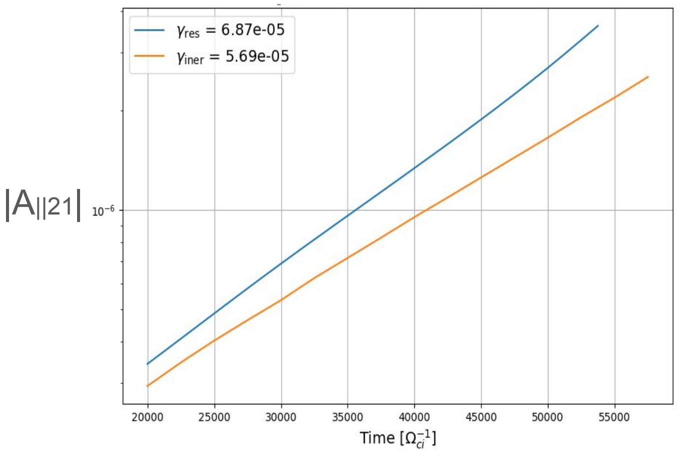
Report on preliminary investigations of tearing instability and nonlinear saturation mechanisms

Study of linear tearing instability

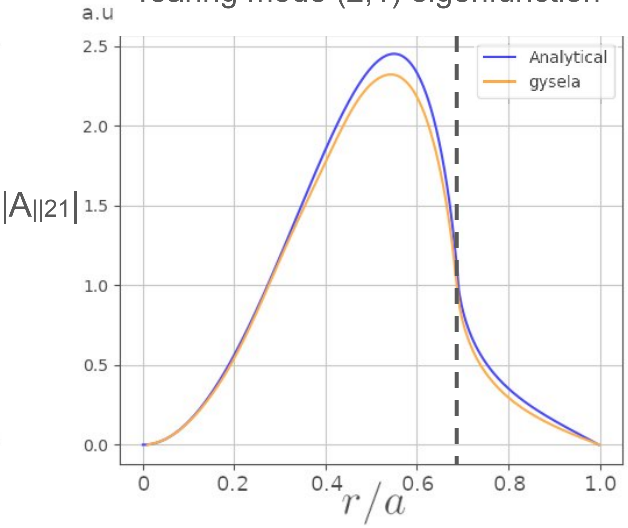
We select a current-gradient unstable on q=2 dominating mode : (2,1)

Flat density/temperature profiles, large aspect ratio (10)

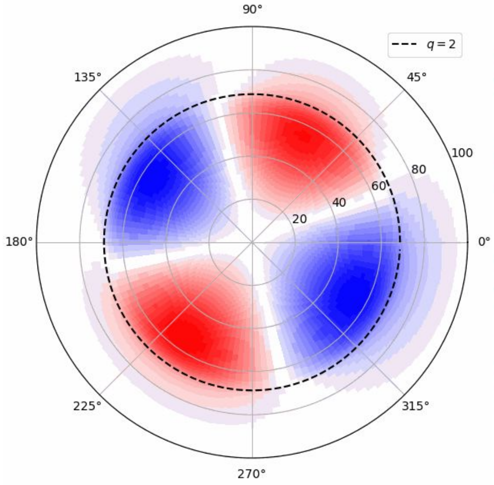
$|A_{||21}|$ linear instability



Tearing mode (2,1) eigenfunction

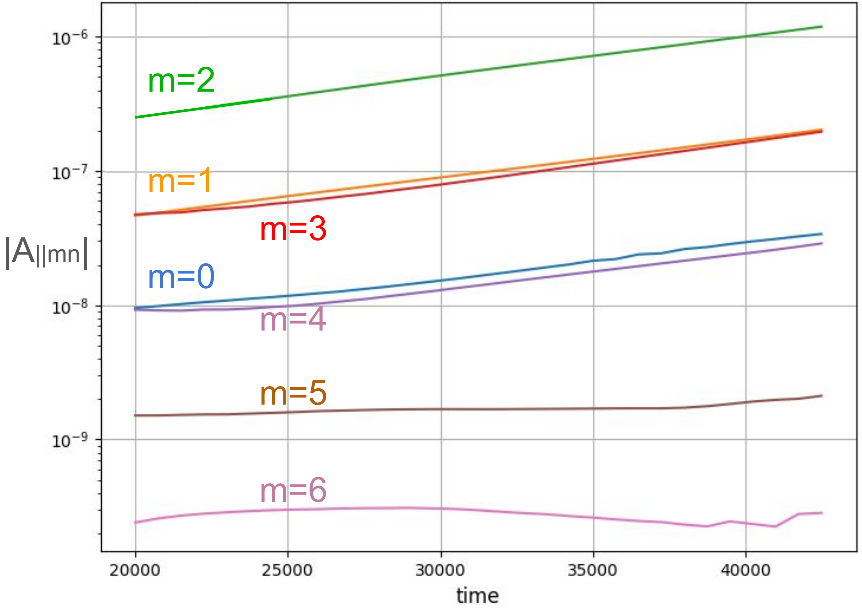


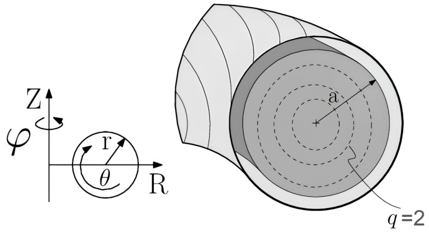
$A_{||}(r,\theta,\varphi=0) - \langle A_{||}\rangle_\varphi$

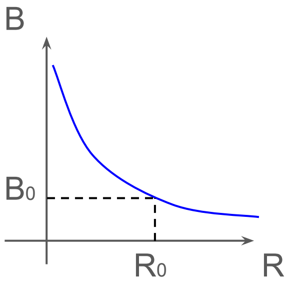


Linear toroidal coupling

$A_{||}$ n=1 modes dynamics







- Toroidicity brings

$$B(\theta) \approx B_0\,(1 - \varepsilon\cos\theta)$$

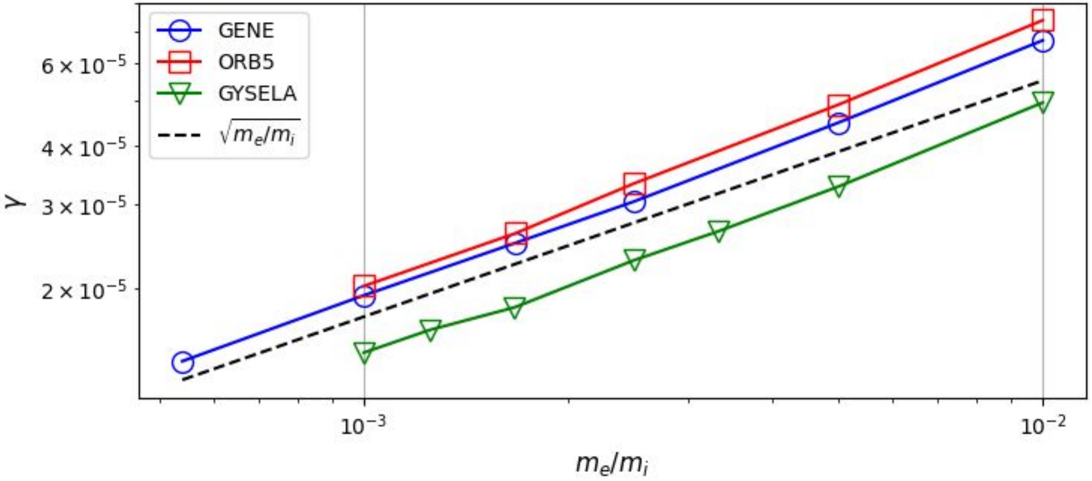
$$\varepsilon = r/R_0 \qquad \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

- Eigenmode sidebands broadening
- Linearly couples any m±1 mode
  γ_{21} driving, ε/2 ratio

Collisionless tearing instability

For collisionless tearing, we expect $\gamma \propto \sqrt{\frac{m_e}{m_i}}$ [Drake & Lee 1977]

Collisionless tearing instability evolution with mass ratio



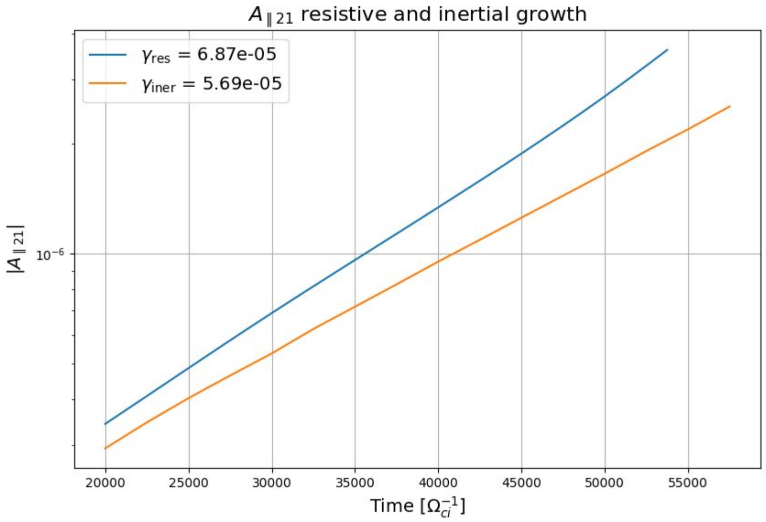
[Jitsuk 2024]

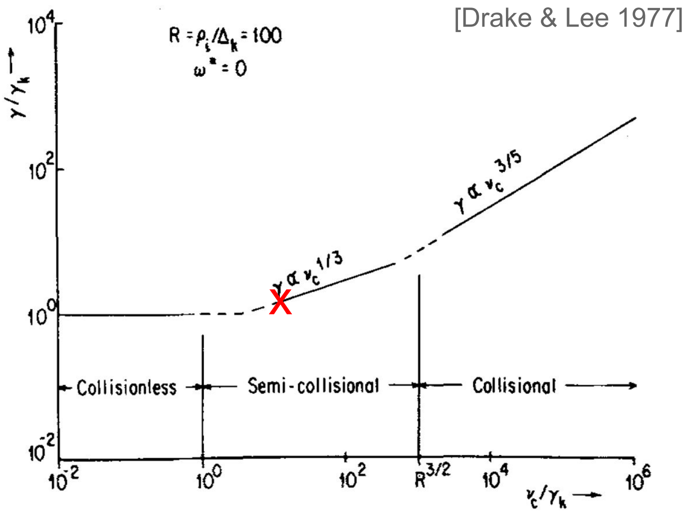
Our results reproduce the characteristic mass-ratio scaling

Discrepancy expected to come from equilibrium current gradient introduction

Resistive tearing instability

collisional case : $\nu_{ei} / \gamma \approx 2$





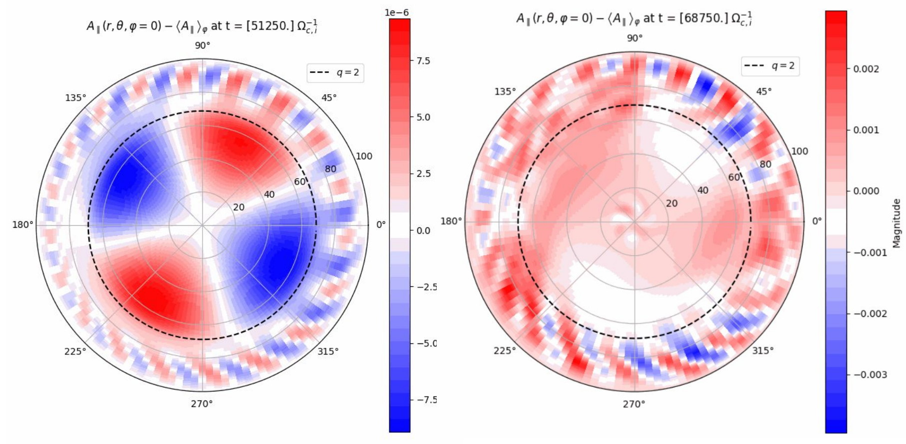
No resistive tearing instability scans however

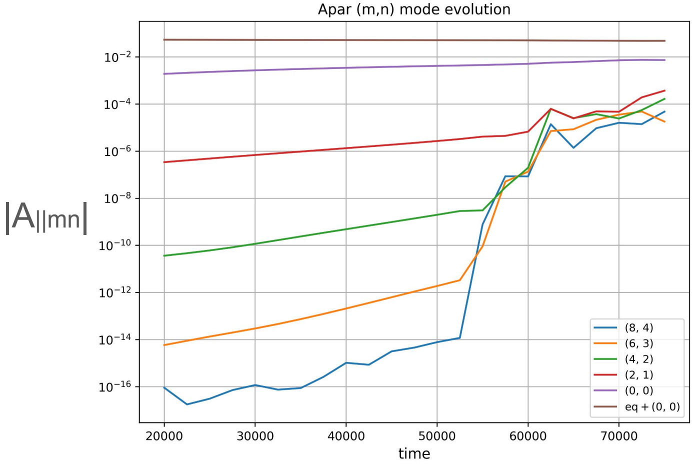
Focus on NL saturation study, we prioritized 2 long simulations dominated by NL effects

Magnetic reconnection saturation simulations

Lost bet, after 3-4 weeks of 64 Adastra nodes simulations (2.5 millions CPU hours)

Our collisional simulation revealed unexpected edge physics approaching NL phase

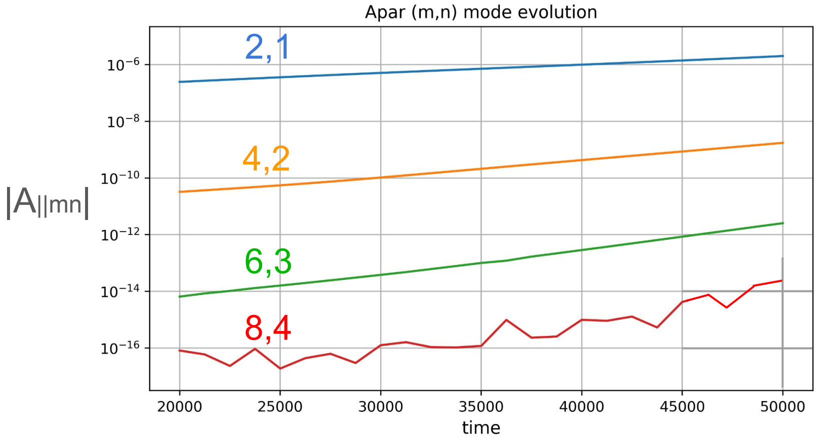




Currently under investigation

Weak non-linear coupling

Large amount of modes in simulations : $n \in [-20,20]$ & $m_{res} - 20 \leq m \leq m_{res} + 20$



(m,n)	(2,1)	(4,2)	(6,3)	(8,4)
γ [Ω_{ci}]	0.69e-4	1.4e-4	2.0e-4	2.8e-4

$\gamma_{2.k,1.k} \simeq k.\gamma_{2,1}$

Resonant harmonic cascade is captured : playing important role in island shape and saturation

Similar results for both collisional and collisionless cases

Equilibrium evolution

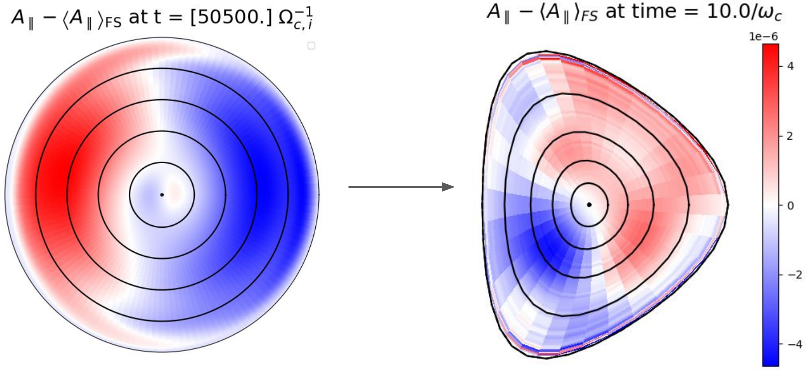
n = 0 perturbation modes in the simulations : associated with axisymmetric background B field

GYSELA electromagnetic **full-f** code : self-consistent evolution of the B configuration

Background configuration evolution tricky to handle : crash with initial version of the code

New Ampère solver to refine initial magnetic configuration → smaller relaxation to an real equilibrium

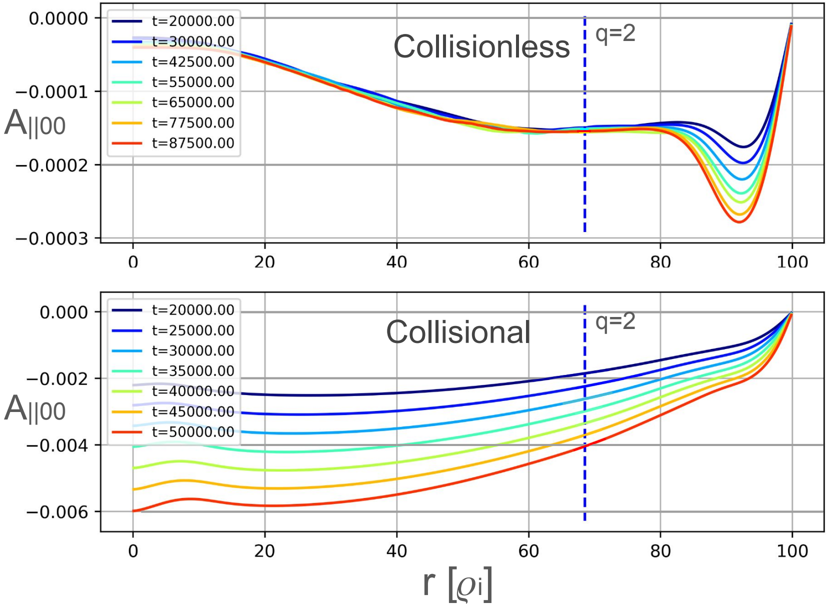
[Obrejan 2021 & Bourne 2022]



Equilibrium $A_{||}00$ evolution

New solver allows us to capture equilibrium relaxation

perturbed $\hat{A}_{|| (0,0)}$ radial profile evolution



- Magnetic relaxation captured except edge build-up
- Resistive diffusion of magnetic equilibrium
- Opening the way for NL profile-flattening effect

Conclusion

Despite ongoing numerical challenges, we now consistently capture

1. resistive and inertial tearing linear instability
2. nonlinear harmonic coupling
3. equilibrium evolution

Laying ground for gyrokinetics studies of :

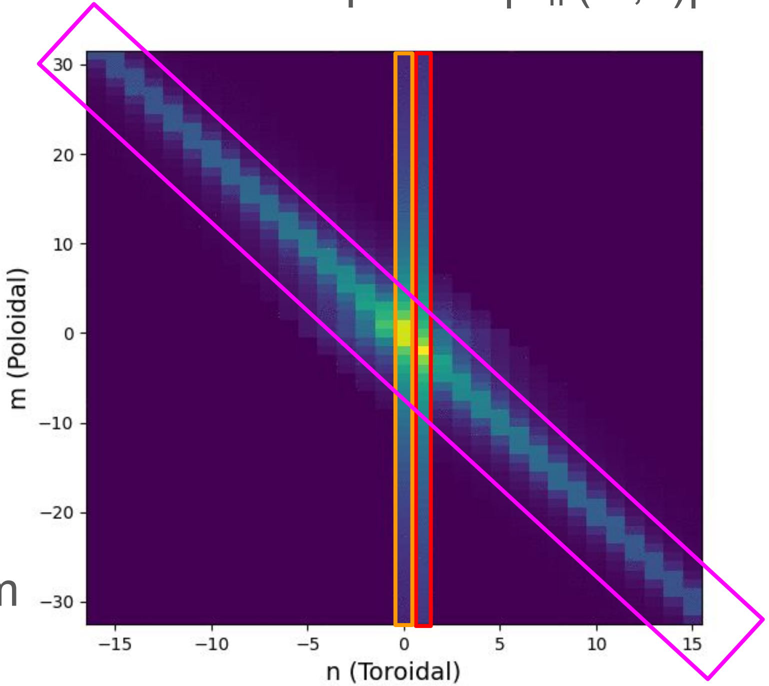
1. **simulation of nonlinear saturation through profile flattening**
2. **neoclassical tearing mode growth**
3. **multi-scale turbulence interaction with self-consistent magnetic islands**

Annexes

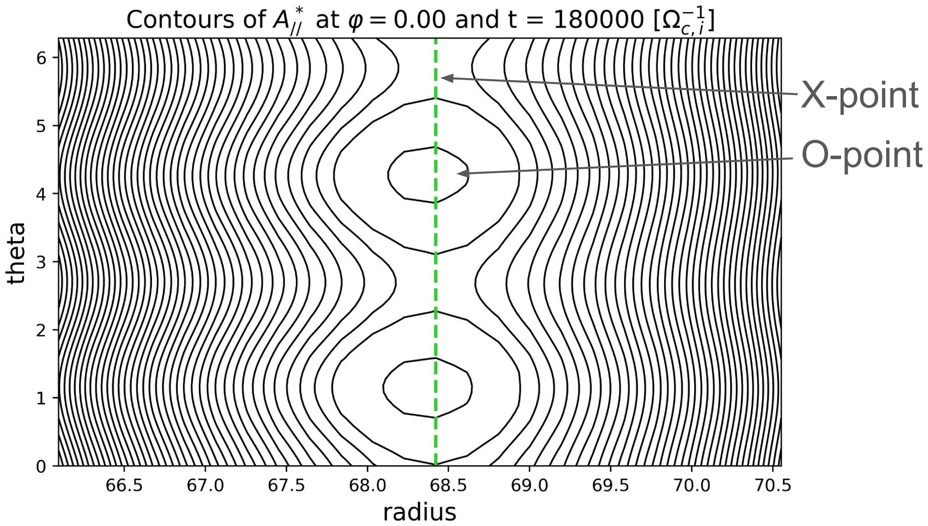
Mode interaction and coupling

- Until now only n=1 physics
  Interesting for linear phenomena
- Resonant cascade q=m/n
  Impact MI saturation and shape
- Equilibrium physics on n=0
  Eq. relaxation, profile flattening
- Algorithm needs to handle all of them
  before describing MI ↔ turbulence

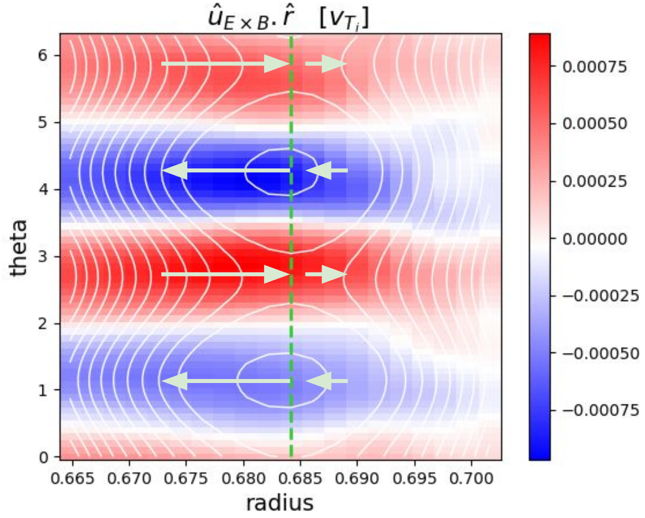
Sketch of expected $|A_{||}(m,n)|$



Magnetic islands and flow structure

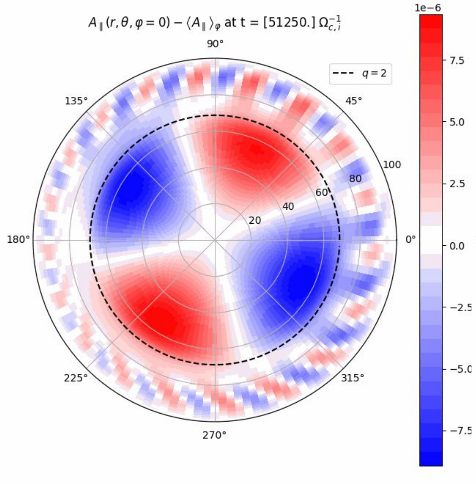


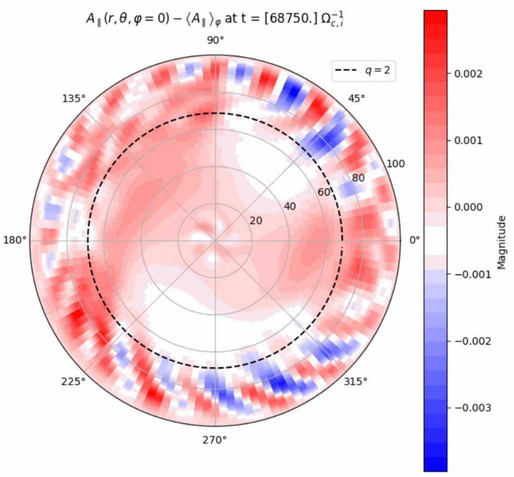
Surrounding flow structure



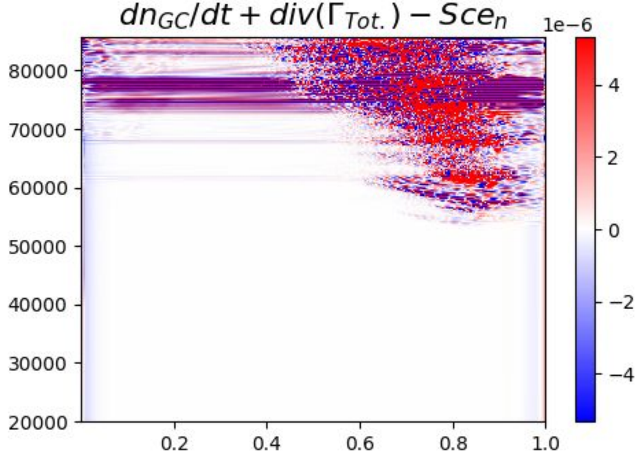
Simulations consistent with the classical picture

Boundary condition and charge conservation

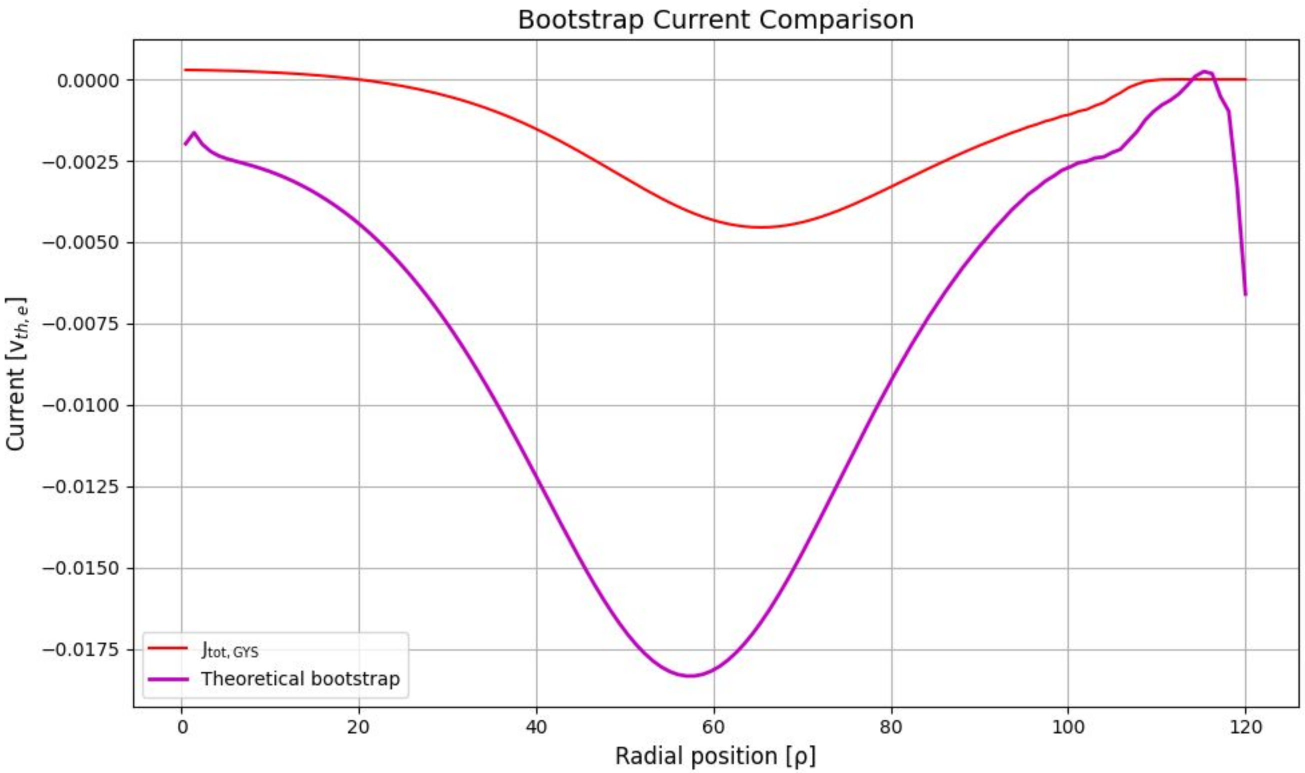




Ion conservation



Bootstrap current



[Sauter 1999]

Electromagnetic gyrokinetic model

Particle trajectories in $v_{\|}$-formulation :

$$\dot{\mathbf{X}} = v_{\|}\tilde{\mathbf{b}}^* + \frac{1}{q_s \tilde{B}_{\|}^*}\mathbf{b} \times \left[\mu \nabla B + q_s \left(\nabla \langle \phi \rangle + \frac{\partial \langle A_{\|} \rangle}{\partial t}\mathbf{b} \right)\right]$$

$$\dot{v}_{\|} = -\frac{1}{m_s}\tilde{\mathbf{b}}^* \cdot \mu \nabla B - \frac{q_s}{m_s}\left(\tilde{\mathbf{b}}^* \cdot \nabla \langle \phi \rangle + \frac{\partial \langle A_{\|} \rangle}{\partial t} \right)$$

requires semi-implicit

$$\tilde{\mathbf{B}}^* = \mathbf{B} + \frac{m_s}{q_s} v_{\|} (\nabla \times \mathbf{b}) + \nabla \times (\langle A_{\|} \rangle \mathbf{b}), \quad \tilde{B}_{\|}^* = \mathbf{b} \cdot \tilde{\mathbf{B}}^*, \quad \tilde{\mathbf{b}}^* = \frac{\tilde{\mathbf{B}}^*}{|\tilde{\mathbf{B}}^*|}$$

With Ampère equation being :

$$-\nabla_{\perp}^2 A_{\|} = \mu_0 \sum_s q_s \int v_{\|} (f_s - f_{s,t=0}) \mathrm{d}v_{\|} \frac{2\pi B}{m_s} \mathrm{d}\mu$$

Mixed-variable scheme

Particle trajectories in $p_{\|}$-formulation : $p_{\|} = v_{\|} + \frac{q_s}{m_s} A_{\|}$

$$\dot{\mathbf{X}} = \left(p_{\|} - \frac{q_s}{m_s} \langle A_{\|} \rangle \right) \mathbf{b}^* + \frac{1}{q_s B_{\|}^*} \mathbf{b} \times \left[\mu \nabla B + q_s \left(\nabla \langle \phi \rangle - p_{\|} \nabla \langle A_{\|} \rangle \right) \right]$$

$$\dot{p}_{\|} = -\frac{1}{m_s} \mathbf{b}^* \cdot \mu \nabla B - q_s \mathbf{b}^* \cdot \left(\nabla \langle \phi \rangle - p_{\|} \nabla \langle A_{\|} \rangle \right)$$

$$\mathbf{B}^* = \mathbf{B} + \frac{m_s}{q_s} v_{\|} (\nabla \times \mathbf{b}), \quad B_{\|}^* = \mathbf{b} \cdot \mathbf{B}^*, \quad \mathbf{b}^* = \frac{\mathbf{B}^*}{|\mathbf{B}^*|}$$

With Ampère equation being : $-\nabla_{\perp}^2 A_{\|} = \mu_0 \sum_s j_{1\|s} - \sum_s \frac{\beta_s}{\rho_s^2} \langle A_{\|} \rangle_s$ skin term

Cancellation problem

Non-physical skin term dominates the Ampère equation → pollutes $A_{||}$

$$-\nabla_{\perp}^{2} A_{||} = \mu_0 \sum_s j_{1||s} - \sum_s \frac{\beta_s}{\rho_s^2} \langle A_{||} \rangle_s$$

To remedy, we decompose $A_{||}$ in a Symplectic and a Hamiltonian part

$$A_{||} = A_{||}^{S} + A_{||}^{H}$$

We choose the Symplectic part as the ideal MHD solution to remove large scales

$$\frac{\partial A_{||}^{S}}{\partial t} = -\nabla_{||} \phi$$

[Mishchenko 2015, Gillot 2020]

Mitigation of magnetic cancellation

$$A_{\|} = A_{\|}^{S} + A_{\|}^{H} \qquad \frac{\partial A_{\|}^{S}}{\partial t} = -\nabla_{\|}\phi$$

The skin term is then reduced to the Hamiltonian part only

$$-\nabla_{\perp}^{2} A_{\|}^{H} = \mu_0 \sum_s j_{1\|s} + \nabla_{\perp}^{2} A_{\|}^{S} - \sum_s \frac{\beta_s}{\rho_s^2} \langle A_{\|}^{H} \rangle_s$$

Effective mitigation for fast Alfvén waves but electrostatic modes brings accumulation