Hybrid fluid-kinetic simulations of tearing mode instabilities in runaway electron beams

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Catalogue

- **Background**
- **Runaway electrons and hybrid model**
- **Verification: 2D & 3D**
- **Nonlinear tearing mode dynamic simulation**
- **2,1 tearing mode with REs**
- **Ongoing work and conclusion**

Background

Runaway electrons

[1]

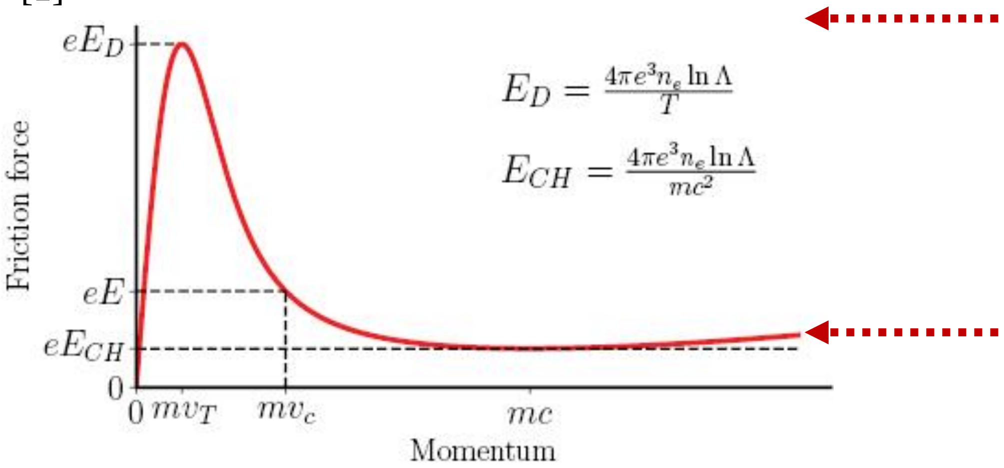


FIG. 1. Momentum-dependent friction force for an electron in comparison with the driving electric field E.

FIG. 2. Melting of a Beryllium tile in JET tokamak due to runaway electron

Runaway electrons (REs):

- Highly **supra-thermal** electrons, which may become a significant or even the dominant current carrier.
- For RE from a Maxwellian tail, when $E > E_{CH}$.
- All electrons enter the runaway regime when $E > E_D$.

[1] Boris N. Breizman et al 2019 Nucl. Fusion 59 083001
URL: https://www.euro-fusion.org/news/detail/tokamak-principle/? (cit. on pp. 25, 27, 42)

Disruptions

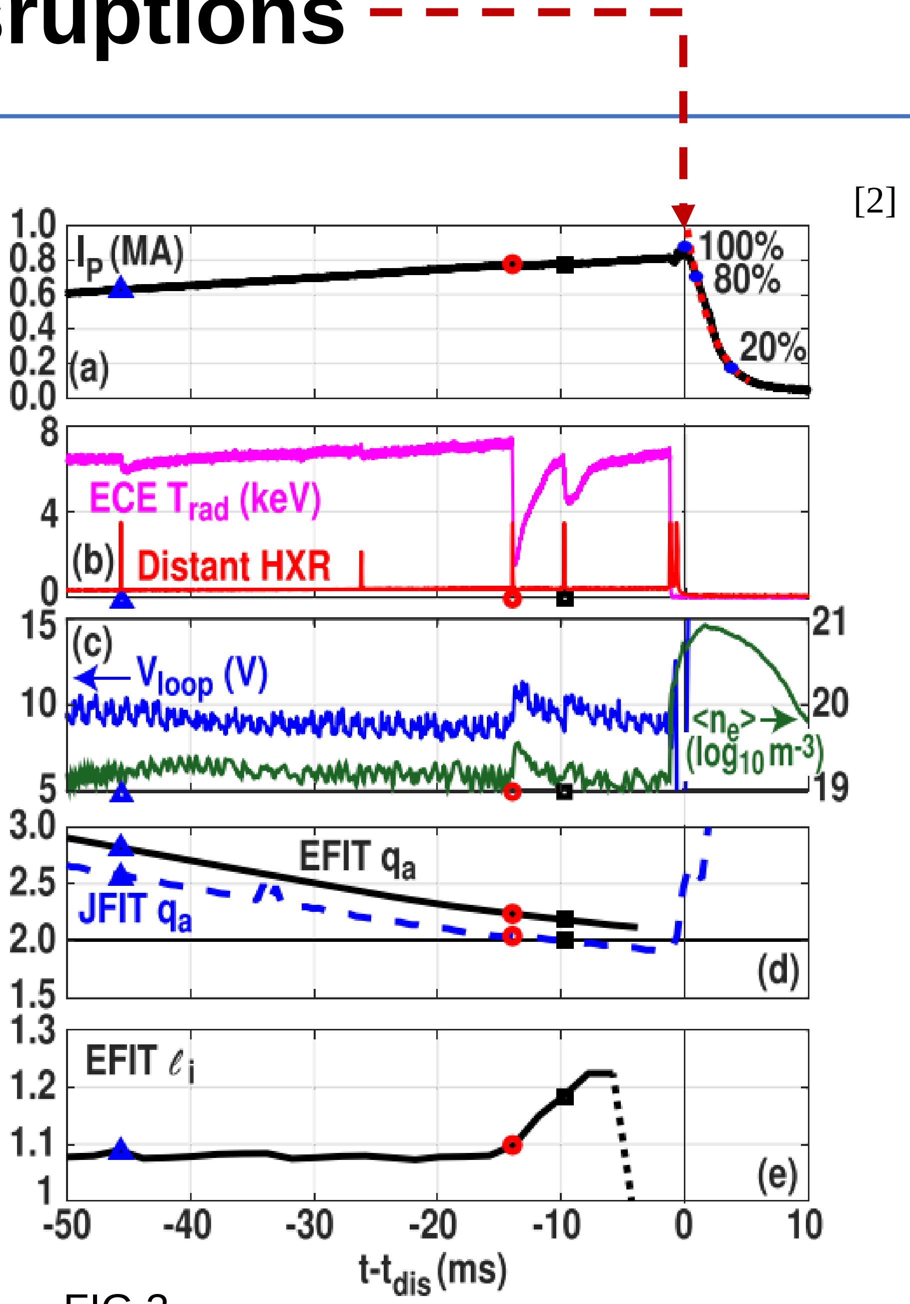


FIG.3.

Pre-disruption

- Enhanced MHD activity
- Formation of runaway **seed**

Thermal quench

- Sudden loss of plasma thermal energy
- Fast temperature drop
- Rapid increase in plasma **resistivity**

Current quench

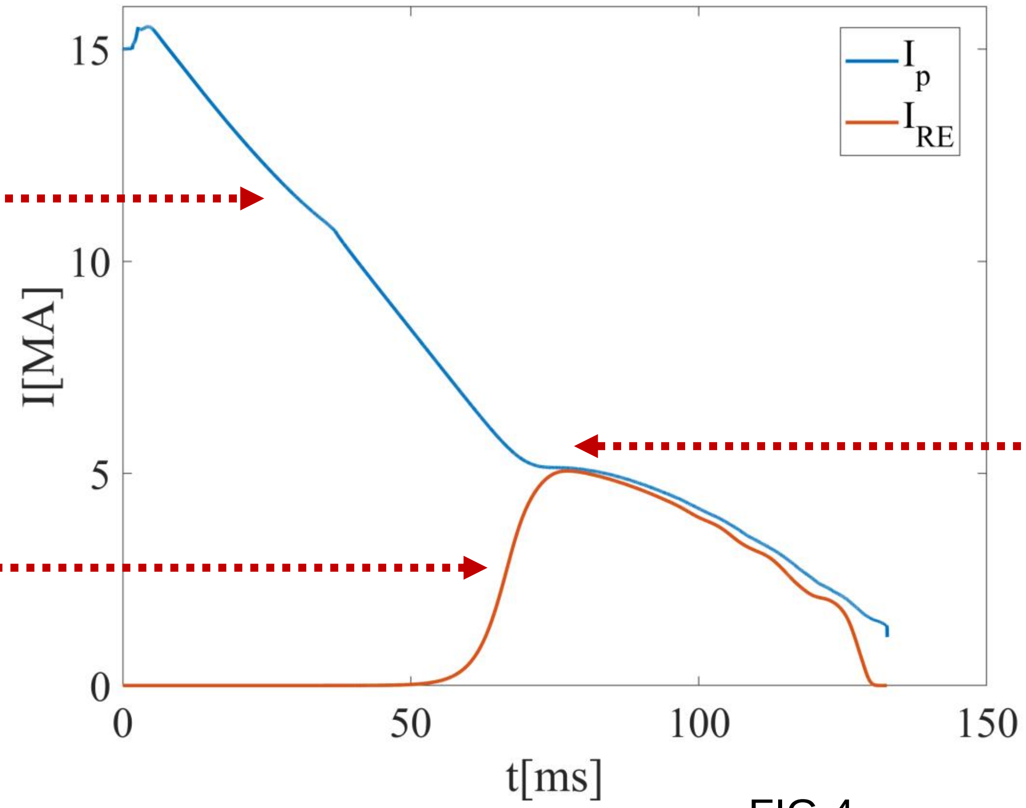
- Decay of plasma current
- Generation of strong **electric fields**
- Formation of **runaway electron beam**

[2] C Paz-Soldan et al 2019 Plasma Phys. Control. Fusion 61 054001.

Runaway electrons avalanche

Majority of I_p carried by I_{Ohm}

Exponential growth of I_{RE}



I_{RE} saturates. Majority of I_p carried by REs.

FIG.4.

RE avalanche gain :

$$g_{RE} = I_{RE}/I_{RE,t=0} = \exp\left(\Delta I_p/I_c\right)$$

$g_{RE} > 10^{16}$ in the tokamak disruptions with I_p=15MA

Hybrid model and resistive instability with REs

Coupling

- My purpose is to establish a **drift-kinetic** coupling model that would aim at **long time scales** interactions between **runaway electrons** and large scale instabilities.

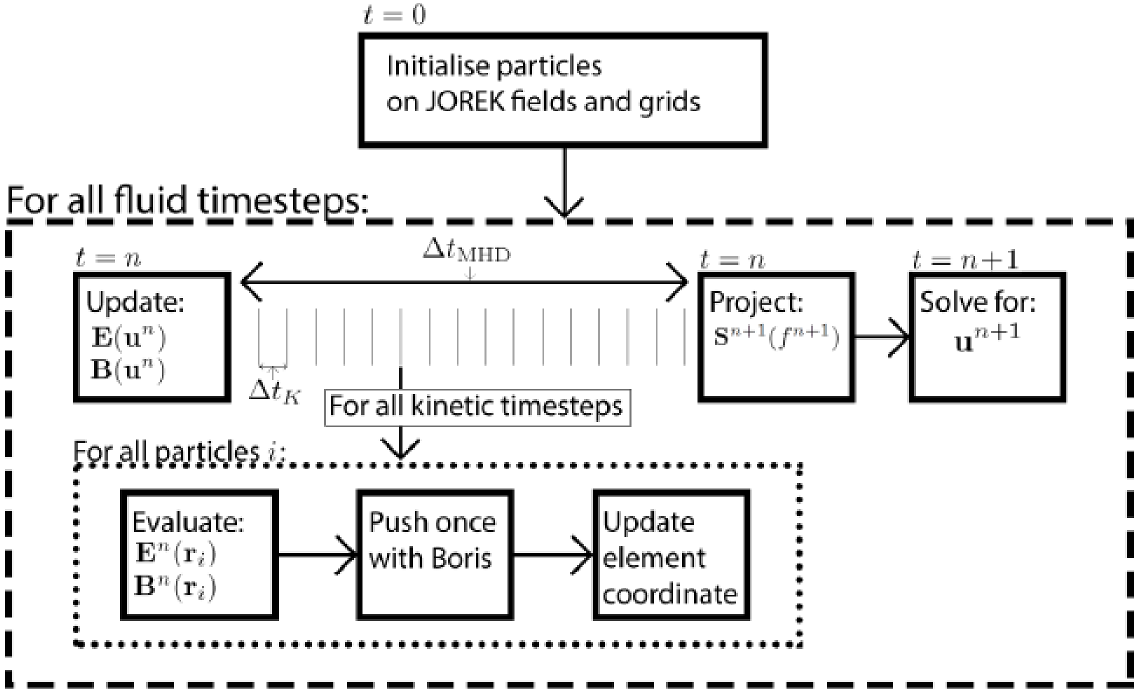


FIG. 5. Flow chat of the kinetic model coupling with MHD model.

JOREK code

- Non-linear MHD code
- Realistic divertor tokamaks
- A lot expansions…reduced & full MHD; relativistic electron fluid & kinetic…..

Resistive instability with REs

- In a **post-disruption** plasma, the current is entirely carried by the runaway electrons
- Cold background plasma: **resistive**.
- RE current: **no collision**, **kinetic** effect.

PiC (Particle in Cell): in JOREK code

- REs simulated kinetically by pushing macroscopic marker particles in the JOREK fields.
- Downside: Markers evolved on small time scales, can be computationally expensive.
- **Advantage**: Phase space information retained and transport accurately captured.
- Both **full-orbit** and **drift/ gyro-kinetic** models have been implemented.

[3]

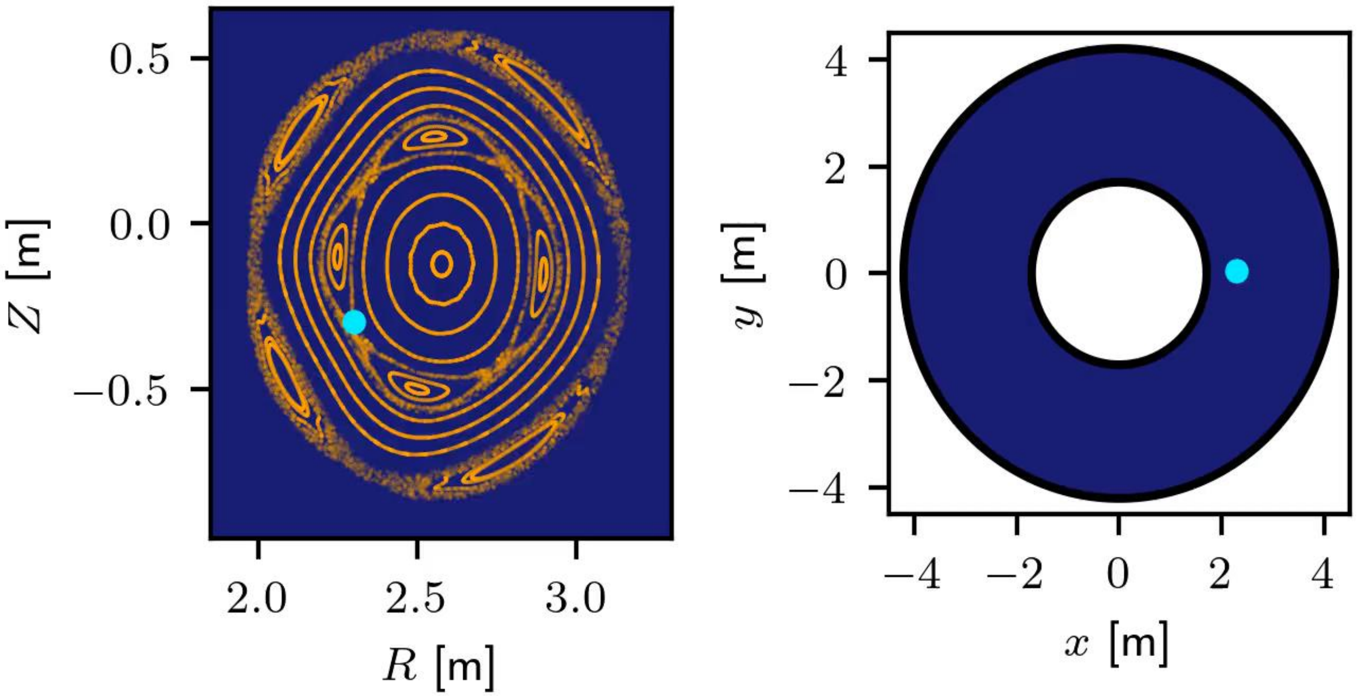


FIG. 6. An example about how REs moving in tokamak.

[3] Hannes Bergström, Runaway Modeling Meeting, 2-6 June 2025.

PiC (Particle in Cell): in JOREK code

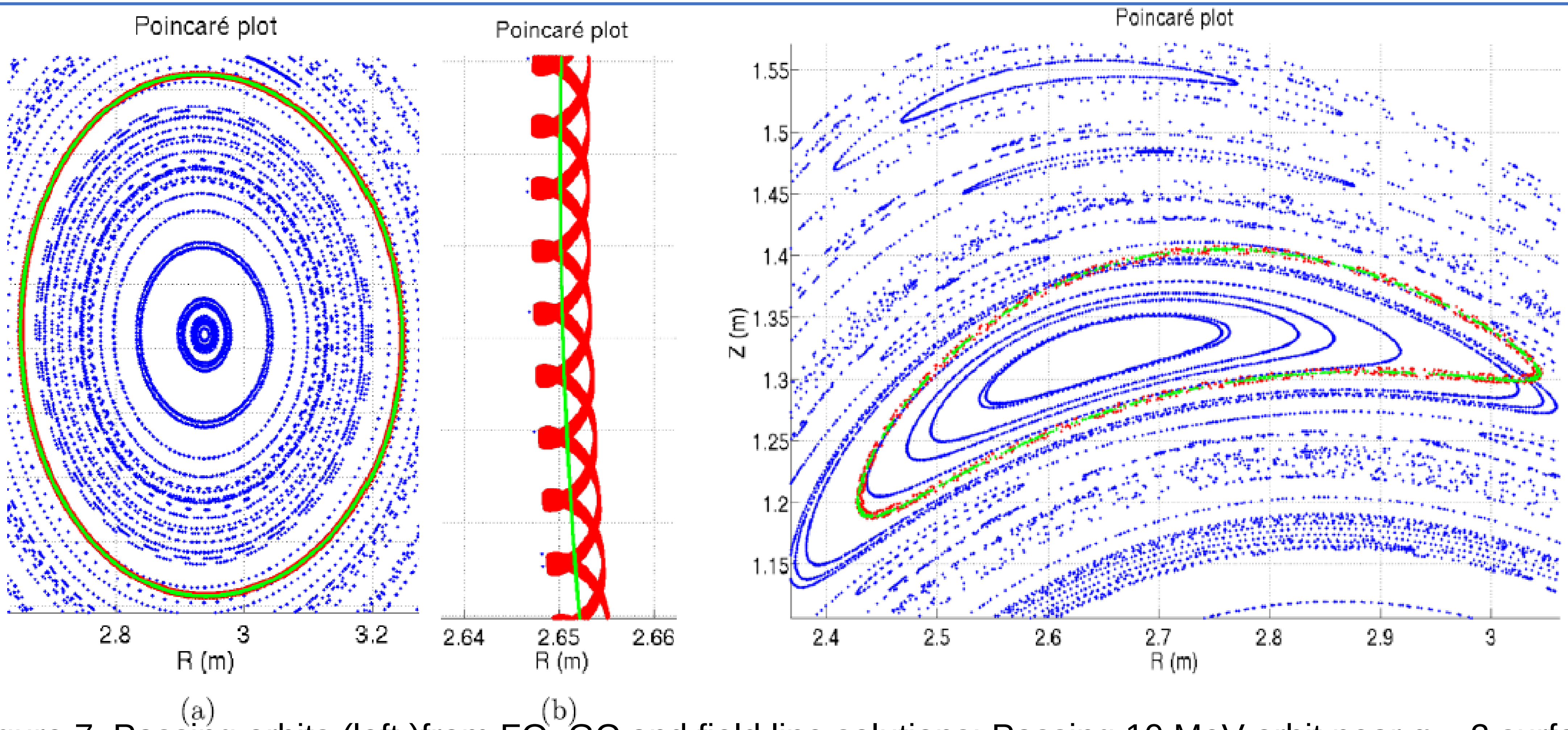


Figure 7. Passing orbits (left)from FO, GC and field line solutions; Passing 10 MeV orbit near q = 2 surface.

[4] C. Sommariva *et al* 2018 *Nucl. Fusion* **58** 016043.

MHD equations coupling with relativistic particles

- **Current coupling scheme**

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\sigma_r \mathbf{E} + (\mathbf{J} - \mathbf{J}_r) \times \mathbf{B} - \nabla p - \nabla\cdot\Pi$$

- **Pressure coupling scheme**

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\sigma_r \mathbf{E}_{\parallel} + \mathbf{J} \times \mathbf{B} - \nabla p - \nabla\cdot\Pi - (P_{r,\parallel} - P_{r,\perp})\kappa - \nabla P_{r,\perp}$$

- **Ohm's law**

$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta(\mathbf{J} - \mathbf{J}_r)$$

- Where **r** denotes the **runaway electrons**. $\mathbf{J}_r$ is the runaway current density, and mainly along the parallel direction.
- A particle distribution by $f(\mathrm{x},\mathrm{v},t) = \sum_{i=1}^{N} w_i \delta(\mathrm{x}_i(t))\delta(\mathrm{v}_i(t))$, where w_i, x_i, v_i are the weight, position and velocity for the i_{st} particle, and N is the total number of markers.

10^7 markers

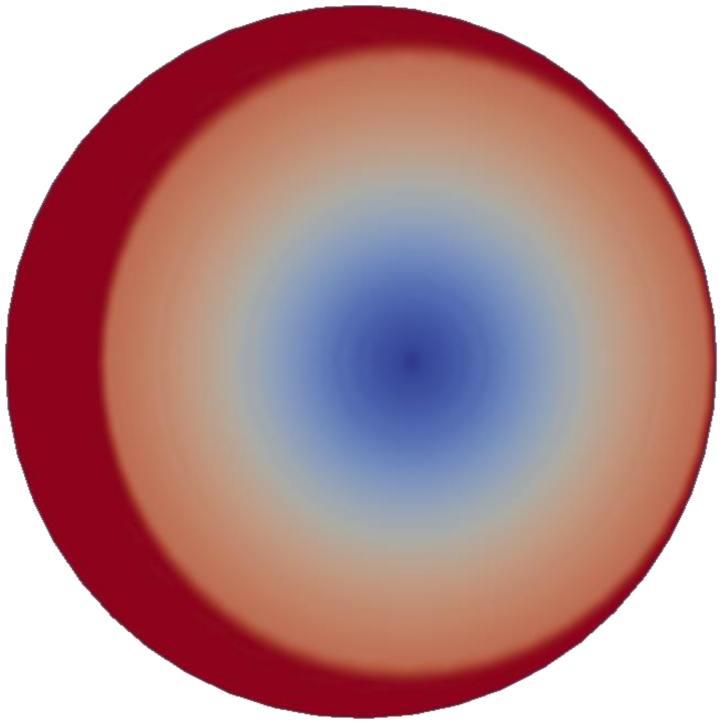
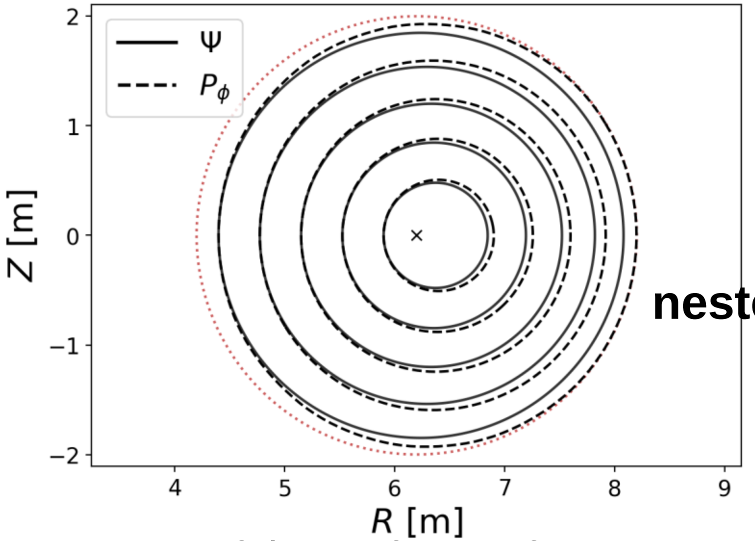


FIG. 8. An example about the current projection from PiC to MHD.

Verification: 2D & 3D

2D Verification: Equilibrium with $J_{total} = J_{re}$



[5]

nested magnetic surfaces

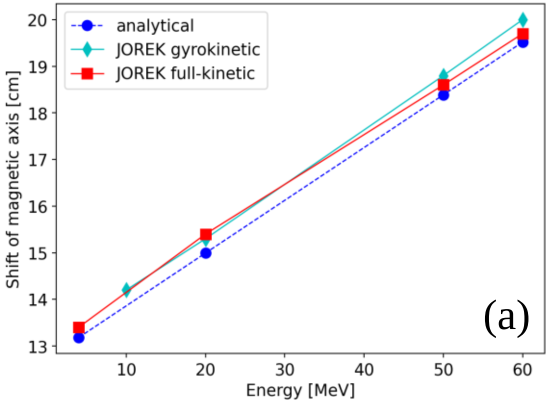
FIG. 9. Contours of the surfaces of constant ψ**(dotted lines)** and RE angular momentum p_ϕ **(solid lines).**

Radial force balance

$p_\parallel \boldsymbol{\kappa} = \boldsymbol{J} \times \boldsymbol{B} - \boldsymbol{\nabla}_\perp p_{th}$ $p_\parallel$: parallel momentum of REs

Grad–Shafranov shift

Refers to the outward displacement of the magnetic axis in a tokamak due to finite plasma pressure p_{th}.



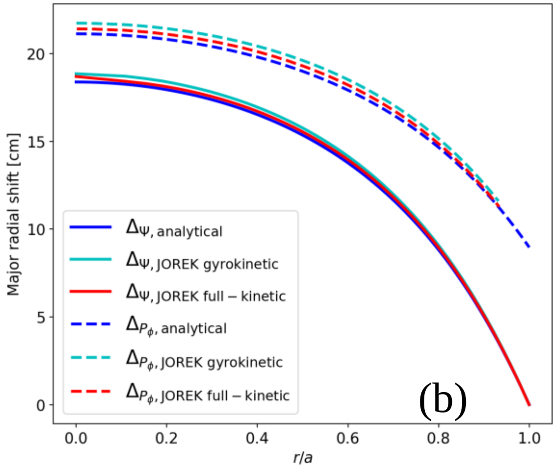
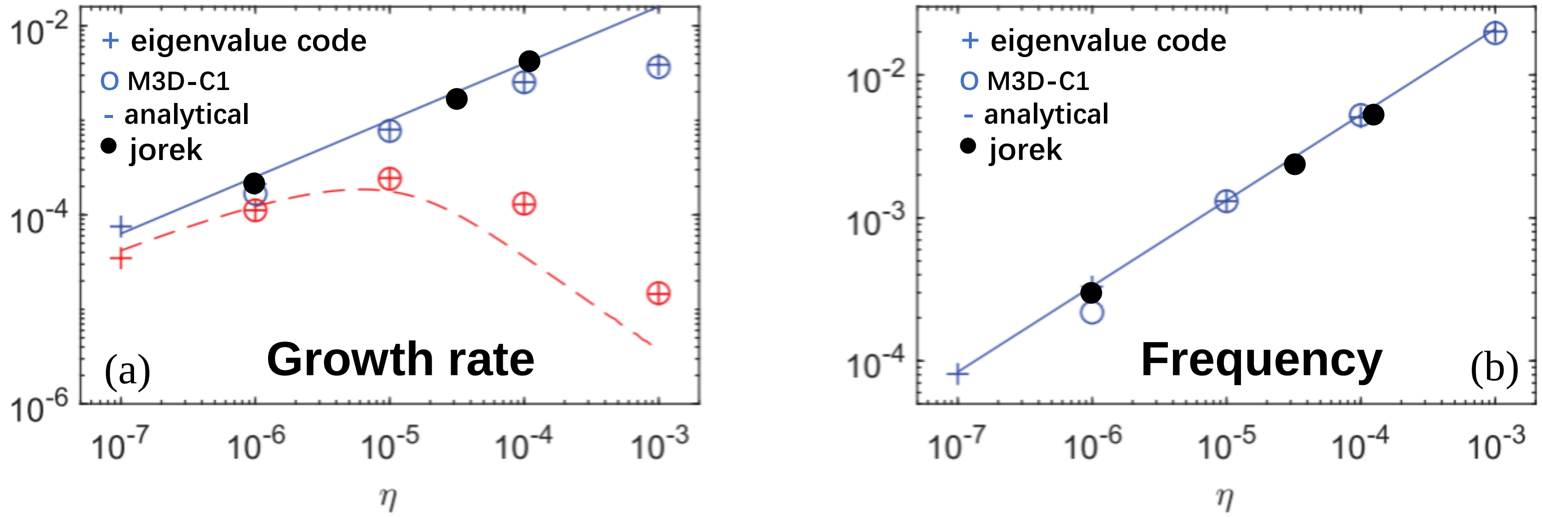


FIG.10. Analytical and JOREK result: (a) shift of magnetic axis varying with **RE energy**; (b) major radial shift profile of 50 MeV RE.

[5] V. Bandaru, M. Hoelzl, *Phys. Plasmas* **30**, 092508 (2023)

3D Verification: linear tearing mode



[6]

FIG. 11. Comparison with analytical result and other methods: (c) Growth rate and frequency for the (2,1) tearing mode in a runaway beam.

$$\frac{\gamma^{5/4}}{\eta^{3/4}k_c^{1/2}}\frac{2\pi\Gamma(3/4)}{\Gamma(1/4)} = \Delta' - i\pi\frac{mJ'_{RE0}}{|k_c|r_s}$$ [4]

- At **high resistivity**: correction term due to local features of current density profile vanishes in the case of RE current.
- A **rotational frequency** appears, related to the RE current gradient .

Consistent with analytical solutions and other numerical results.

[6] C. Liu, *et. al.*, *Phys. Plasmas* **27**, 092507 (2020).

(2,1) current width: linear

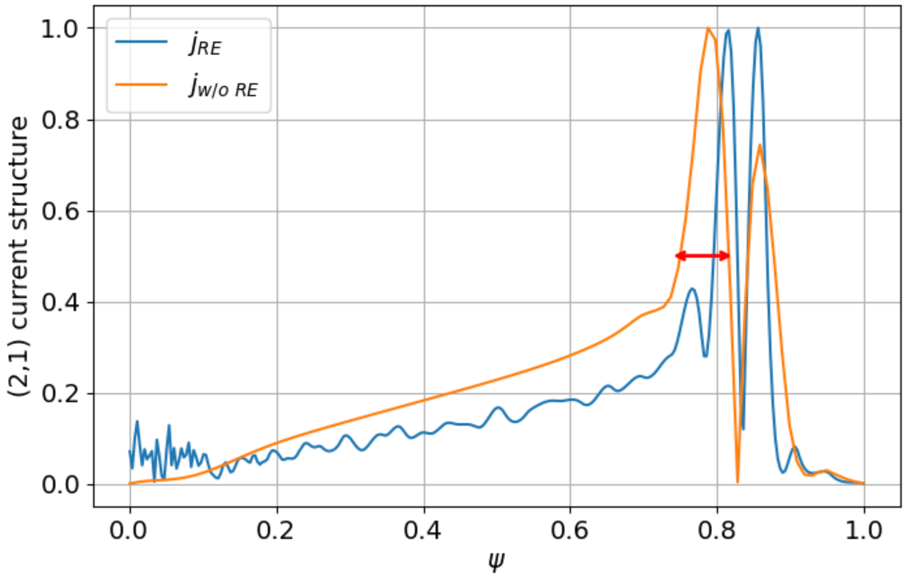


FIG.12. (2,1) current structure profile in the **linear phase**

Resistive layer $\delta_1 = \frac{\gamma^{1/4}\eta^{1/4}}{k_c{}^{-1/2}}$ $k_c = \frac{nq'}{Rq}$

Sublayer $\delta_2 = \frac{\gamma v_A}{k_c v_r}$

$v_A = \frac{B_0}{\sqrt{\mu_0 \rho}}$ is Alfvén speed ~ 10^6-10^7 m/s.

$\delta_2 \ll \delta_1$

- **(2,1) current width with RE is narrower in linear phase (sublayer).**

Nonlinear tearing mode dynamic simulation

Saturated island width

Tearing mode criterion

$$\Delta' = \left[\frac{1}{\psi}\frac{d\psi}{dx}\right]_{x\to0^+} - \left[\frac{1}{\psi}\frac{d\psi}{dx}\right]_{x\to0^-}$$

$\Delta' > 0$ unstable

$\Delta' < 0$ stable

[7]

Saturated island width

$$w = -\frac{1}{b}\frac{\Delta'}{0.272} \quad \text{with runaways,}$$

$$w = -\frac{1}{b}\frac{\Delta'}{0.411} \quad \text{without runaways.}$$

W_re~**1.511**W_w/o

W_re: island width with REs
W_w/o: island width without REs

[7]

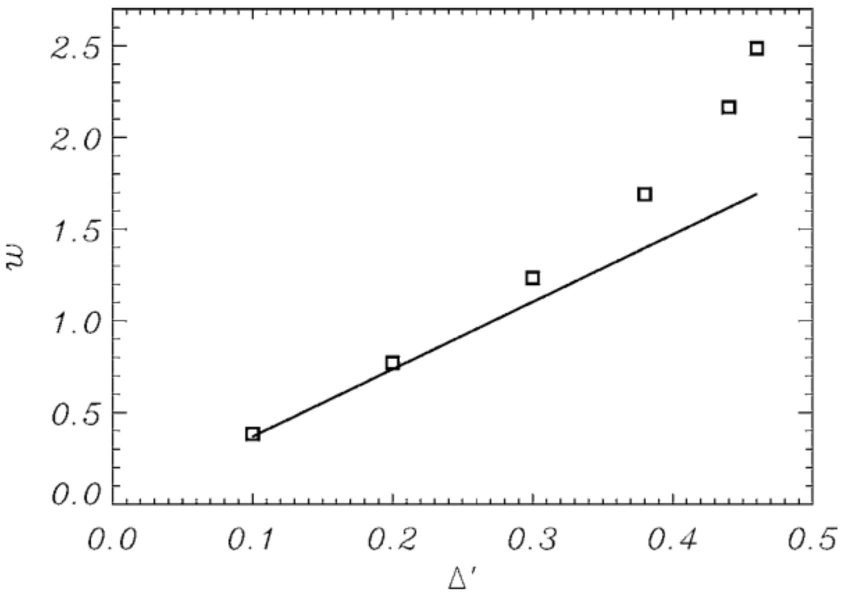


FIG. 13. Saturated island width vs **Δ'**. Note that the slope becomes infinite at **Δ'**~0.44

- **The prediction is under the condition w and $\Delta' \ll 1$;**

[7] P. Helander, *et. al.*, *Phys. Plasmas* **14**, 122102 (2007).

Saturated island width

$\Delta' = 0.2$

W_re~**1.7** W_w/o

$\Delta' = 2.0$

W_re~**2.6** W_w/o

1.5

- **Deviation in $\Delta' = 2.0$ case due to multi-mode coupling.**

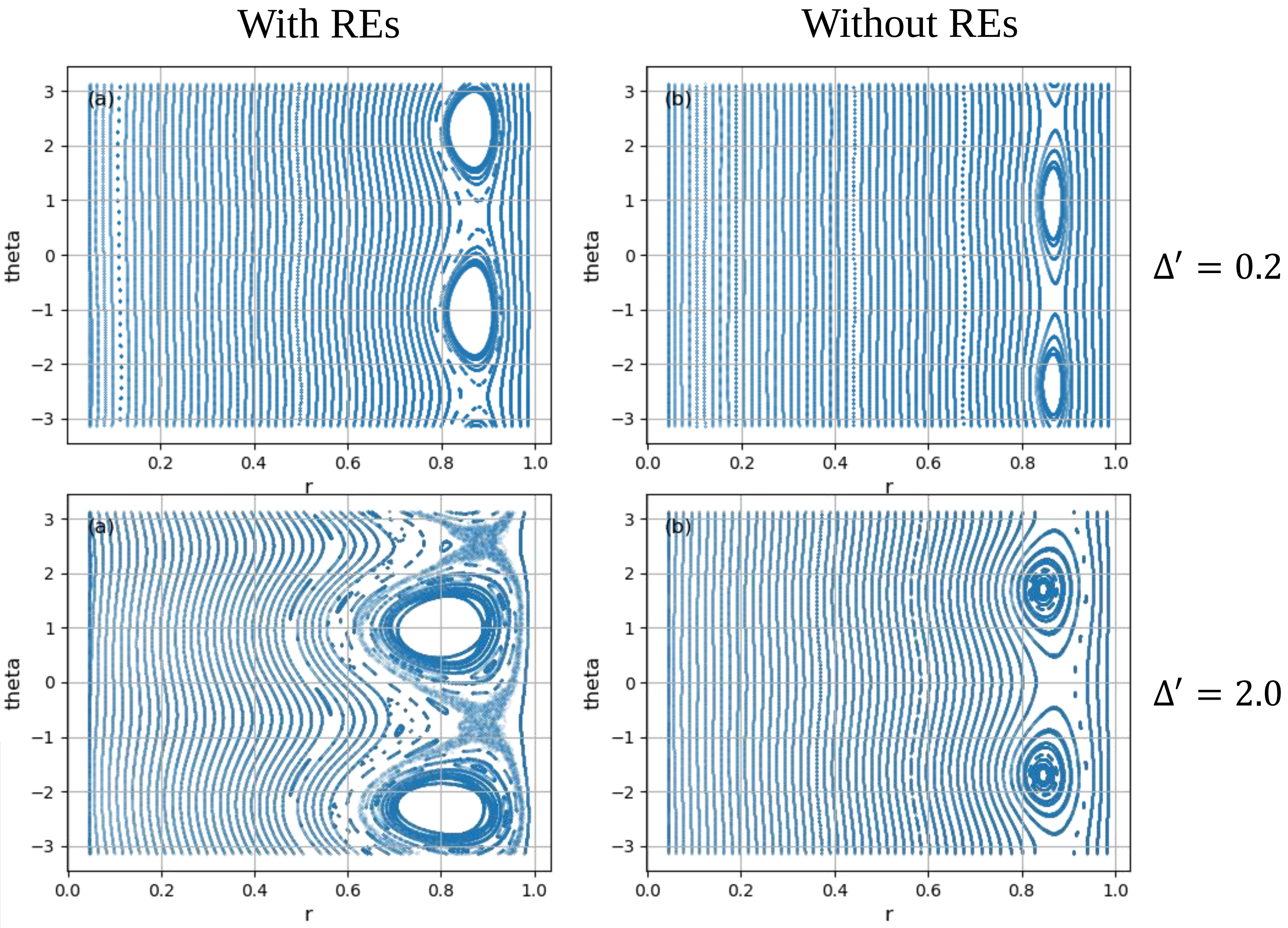


FIG. 14. Poincare plots about saturated island.

(2,1) current width: nonlinear

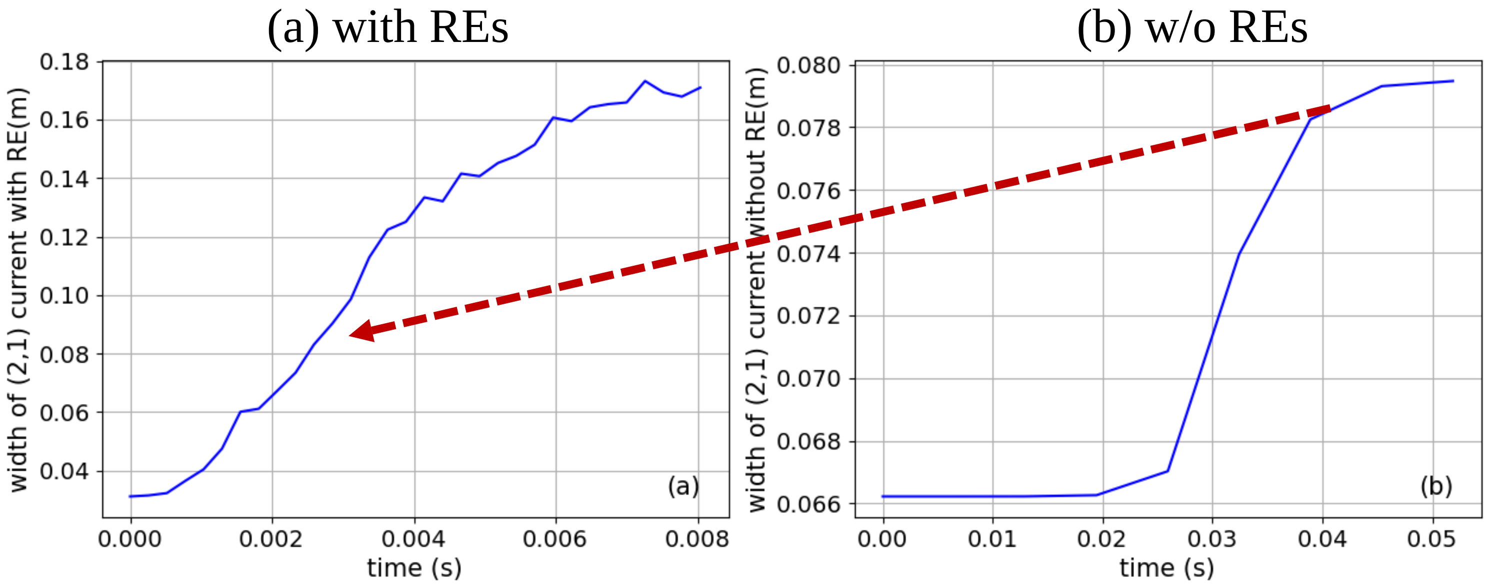


FIG. 15. Time evolution of (2,1) current width

- **(2,1) current width with RE is wider in nonlinear phase due to different response mechanisms.**

Energy evolution

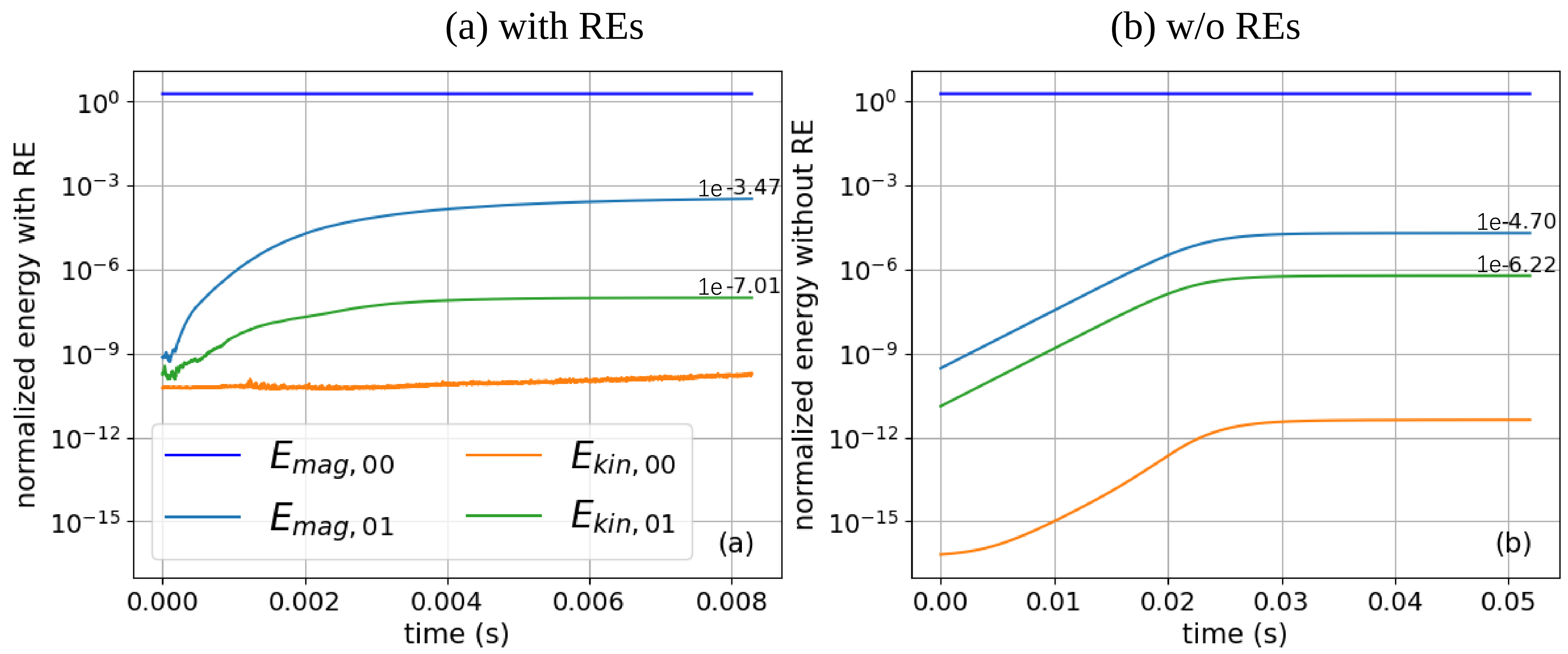


FIG. 16. Energy evolution.

- **n=1 magnetic energy with RE is one order larger than w/o RE.**
- **n=1 kinetic energy with RE is one order smaller than w/o RE.**

Comparison about TM growing process

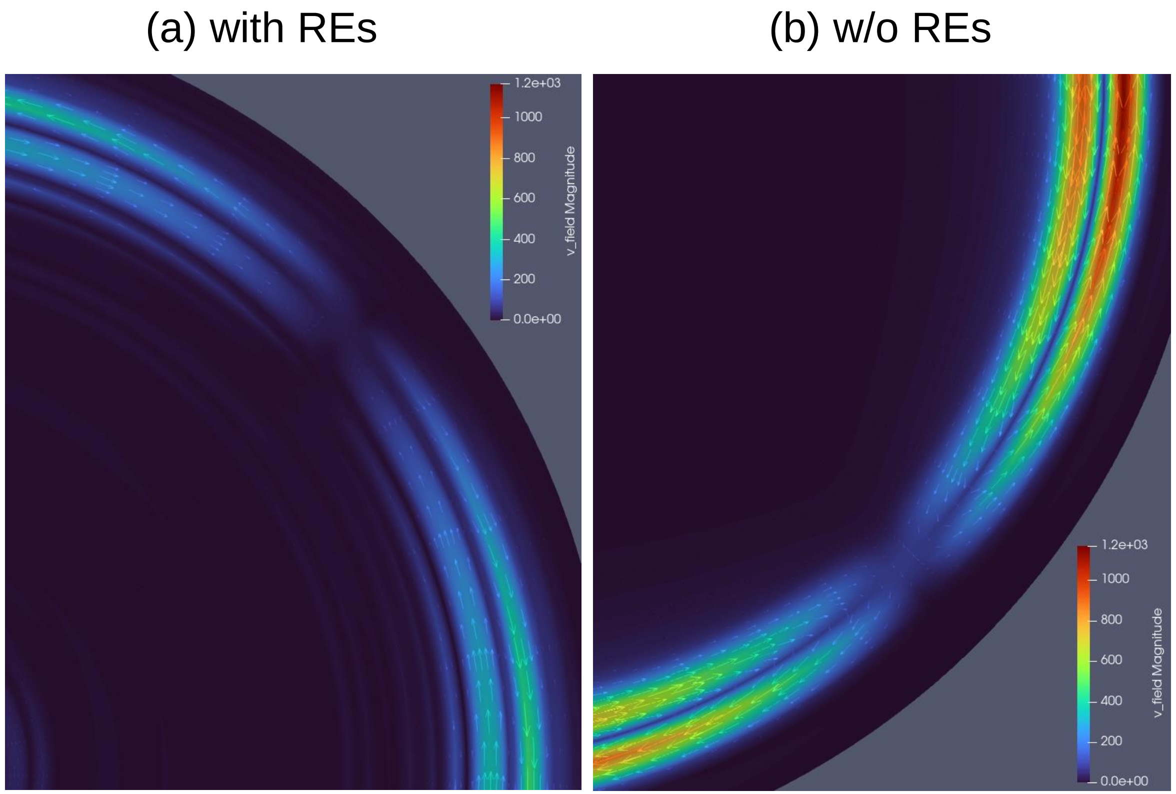


FIG. 17. Reconnection flow: velocity field and vector arrows around the x-point of islands in the $\phi = 0$ R-Z plane.

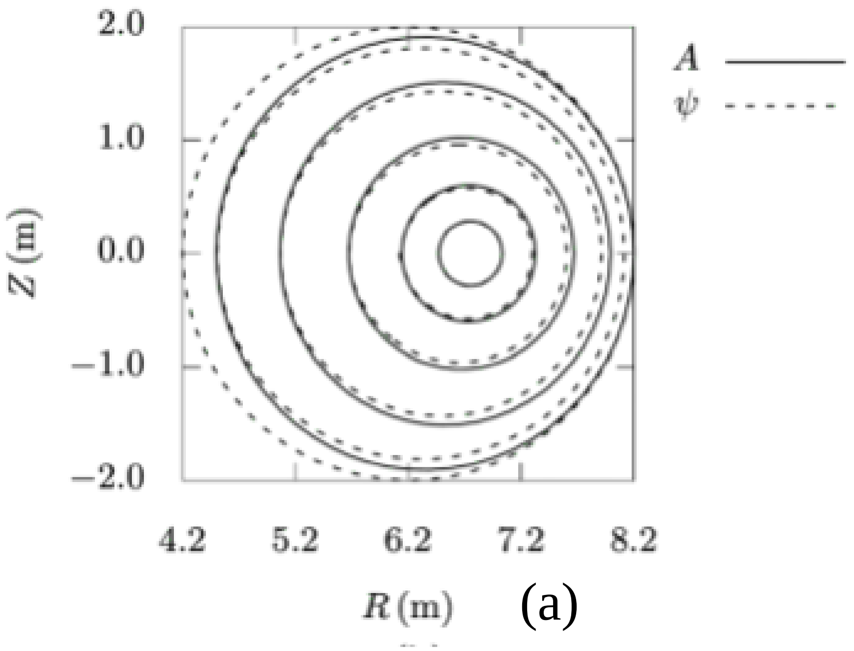
- **In a RE case: (2,1) RE current responds to perturbations immediately in light speed time scale, the magnetic flux within the island increases, which in turn affects the trajectory of REs.**
- **The momentum bulk plasma is decoupled from the evolution of magnetic fields.**

$$\rho_b\left(\frac{\partial \boldsymbol{u_b}}{\partial t} + \boldsymbol{u_b}\cdot\nabla\boldsymbol{u_b}\right) = -\sigma_r\boldsymbol{E} + (\boldsymbol{J}-\boldsymbol{J_r})\times\boldsymbol{B} - \nabla p_b - \nabla\cdot\overleftrightarrow{\boldsymbol{\Pi}}_b - \boldsymbol{R_r} + \boldsymbol{S_{u_b}}$$

Ongoing work and conclusion

Effect of shift on linear TM instability

r = 1 m, R = 10 m, B_0 = 0.3 T, 2MeV RE



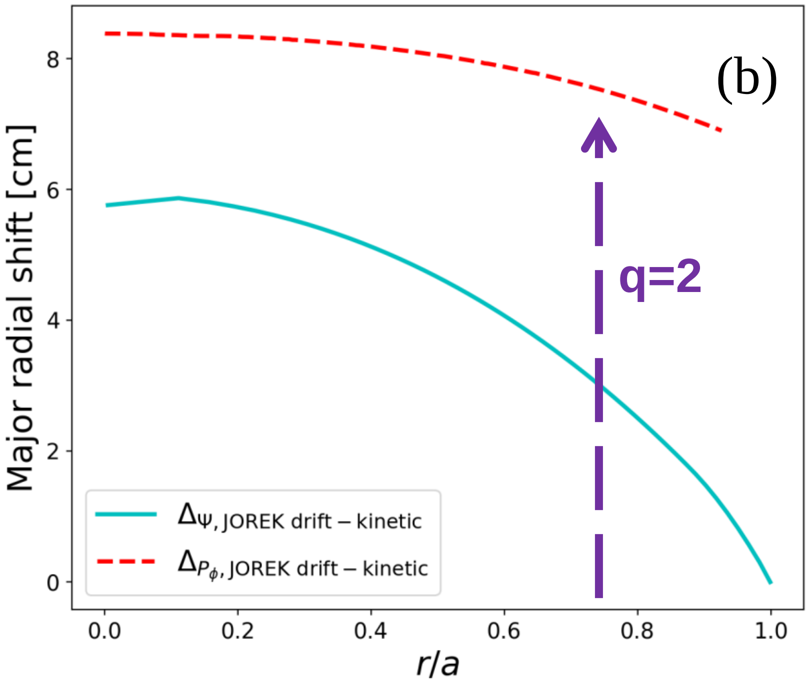


FIG. 18 (a) example about the shift[1], (b) radial shift profiles about flux surface (blue) and particle orbit (red) of 2 MeV RE.

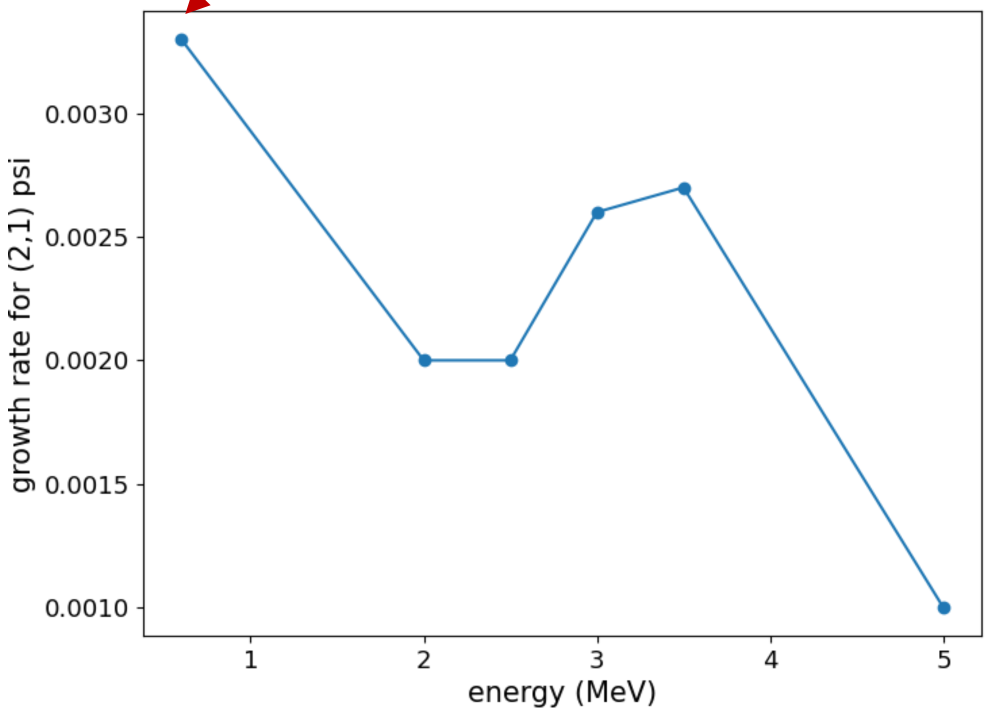
$p_{\parallel}\boldsymbol{\kappa} = \boldsymbol{J} \times \boldsymbol{B}$ eq.(1) **Parallel momentum of REs, without thermal pressure**

$$\Delta^{\mathrm{RE}}(a) = -\left(\frac{\gamma_m v_{\parallel}}{e}\right)\frac{a}{R_0}\frac{1}{B_{\theta 0,a}} \quad \text{eq.(2)}$$

[1] V. Bandaru, M. Hoelzl, *Phys. Plasmas* **30**, 092508 (2023)

Simulation a. 600keV->2MeV

No shift here!



- **Simulation setup -- increasing RE energy and reaching a new equilibrium: linear TM instability in a RE beam**

- **TM growth rate is changed a lot.**

FIG. 19 Comparison in a RE beam with different energy:: growth rate

Another peak away from the rational surface (located at the q2 plus relative shift)

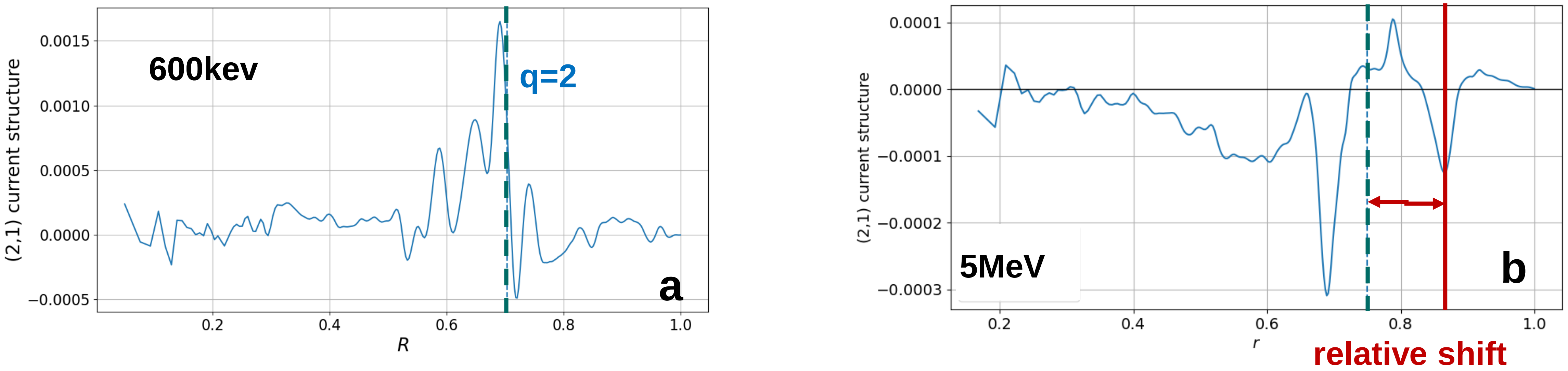


FIG. 20 (2,1) current structure profile in **noshift** RE beam (a) and **10cm-shift** RE beam (b)

- **2,1 current will have an additional structure around the red vertical line, due to runaway orbit shift.**
- **Two parts: one will contribute to MHD instability, the other is due to RE movement.**

Conclusion and outlook

- **Developed and verified hybrid drift kinetic-fluid model**
- **Carried out nonlinear TM dynamic simulations in RE beams.**

- **To investigate the RE energy effect on linear/nonlinear TM.**
- **To study the cases when the relative shift is less than mhd instability scale; more than mhd instability scale.**

Thank you!